Proportions and Reasoning

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Math 600: Math Modeling and Algebraic Thinking
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Big Ideas

- *Proportional Reasoning*
- *Ratios*
- *Solving multi-step equations involving proportions*
- *Problem Solving*

Standards of Learning for Grades 5-8

- G6.1 - The student will describe and compare data, using ratios, and will use appropriate notations, such as, a to b, and a:b.
- G7.4 The student will solve single-step and multistep practical problems, using proportional reasoning.
- G8.3 The student will a) solve practical problems involving rational numbers, percents, ratios, and proportions; and b) determine the percent increase or decrease for a given situation.
- G7.12 The student will represent relationships with tables, graphs, rules, and words.

Standards of Learning for Algebra

- A.1 -The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.
- A.4- The student will solve multistep linear and quadratic equations in two variables, including

  d) solving multistep linear equations algebraically and graphically;
  f) solving real-world problems involving equations and systems of equations.

- A.1.7 - The student will investigate and analyze function (linear and quadratic) families and their characteristics both algebraically and graphically, including a) determining whether a relation is a function; b) domain and range; c) zeros of a function; d) x- and y-intercepts; e) finding the values of a function for elements in its domain; and f) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.

Process Goals

This family of problems focuses on the foundational skill of proportional reasoning at several different levels. We selected problems that required problem solving, reasoning, communication, connections and representations.

Problem solving: We selected problems that varied in complexity to meet the needs of the different levels that we each teach. Each of our problems represents a real-life situation and requires students to use appropriate strategies to find solutions.

Reasoning: Students will be required to learn and apply proportional and spatial reasoning to reason and explain their solutions. They will apply inductive and deductive reasoning in order to justify the validity of their solutions and reasoning.

Communication: Students will explain their thoughts clearly and precisely with one another through the use of mathematical vocabulary to deepen and clarify their thinking. Students will listen to one another and collaborate in order to arrive at a solution.

Connections: Students will be required to collaborate and combine skill sets to arrive at a solution. Each of our tasks requires students to draw from several different topics in mathematics in order to figure out the problem.

Representations: Students will respect and connect several different representations of similar ideas. These representations include graphs, numerical, algebraic, verbal and physical representations.

Environmental Club - Task 1 – Beth Lamy

The Environmental Club at school attends an annual community clean-up event. They have recycling games. A team is assigned an area of land that is scattered with litter. The goal is for a pair of participants to clean up the area in the fastest time possible.

Tammy, working alone, could clean one-half the area in one hour. Her partner Melissa, working alone, could clean one-third of the area in one hour. During the contest when they work together, how long will it take them to clean the area? Explain how you found your solution.

Basic:

Teams of 2 have been formed to clean an area. The goal is for a pair of participants to clean up the area in the fastest time possible.

Frank, working alone, could clean one-third of the area in one hour. Siobhan, working alone, could also clean one-third of the area in one hour. During the contest when they work together, how long will it take them to clean the area? Explain how you found your solution using pictures, words, tables, graphs, and/or symbols.

Extension:

Teams of 3 have been formed. The goal is for a pair of participants to clean up the area in the fastest time possible.

Katelyn, working alone, could clean one-fourth of the area in one hour. Kevin, working alone, could clean one-half of the area in one hour. Bridget can clean one-third of the area in one hour. During the contest when they work together, how long will it take them to clean the area? Explain how you found your solution.

A fourth person joins the team after they have been working for 30 minutes. Mara can clean one-fifth of the area in 20 minutes. How long will it take the clean the area?

Robyn and Josh - Task 2 – Joann Leme

Robyn can mow the lawn in 2 hours, and her brother Josh can do it in 3 hours. If they work together, each with a mower, how many minutes will it take them to mow the lawn?

- Accommodations or modifications teachers can make to reach students who are below the intended level of this task – The numbers could be changed such that the relationship between them is more easily examined.

Robyn can mow the lawn in 2 hours, and her brother Josh can do it in 4 hours. If they work together, each with a mower, how many minutes will it take them to mow the lawn?

- Suggestions for extensions – The questions could be modified for a more challenging problem.

Robyn can mow the lawn in 2 hours, and her brother Josh can do it in 3 hours. When Robyn and Josh are halfway through mowing the lawn, their neighbor decides to come and help them with his riding mower because he sees that a storm is quickly approaching. If he can mow the entire lawn in 30 minutes, how long will it take them all working together to finish the job?

Candle Comparison - Task 3 – Keira Godwin

There are two popular stores that sell expensive candles at the mall. Candles R Us sells a candle that will burn for 4 hours. Candle-mania sells a candle that will burn for 5 hours. After how many hours will one candle be 3 times the size of the other?

Extension: When will the candles be the same size? Defend your reasoning.

Crazy Clocks Task 4 - Jacqlene Pelletier

A broken clock that loses 12 minutes every hour is set at 12:00 noon at the same time as a normal clock has its time set at 12:00 noon on January 1, 2016. When the broken clock reaches 12:00 midnight, what will the normal clock read?

When will the broken clock read the correct time again, how many hours? How many days is it behind?

"What time does the broken clock read in exactly one week? One month? One year?

Additional problem:

After a silly mistake on a math competition, you find yourself at the Moderators' Alligator Association, determined to wreak havoc on the one true enemy: clocks. You decide to set all the clocks a minute forward- that'll show them!

There are two buttons on the clock:  and  . When you hit the  button, the time displayed on the clock jumps ahead by 7 minutes. When you hit the  button, the time on the clock goes back by 7 minutes.

One note- I accidentally dropped all of the clocks earlier, so the hour never changes. If the time is 6:56 and you press  , the clock will show 6:03.

Well don't just sit there! It's 6:03 now. What is the fewest number of times you can press either button to get the clock to read 6:04? Assume no time passes as you calculate or press buttons."

Environmental Club (Version 2) - Task 5 – Nicole Novak

The Environmental Club at school attends an annual community clean-up event. They have recycling games. A team is assigned an area of land that is scattered with litter. The goal is for a pair of participants to clean up the area in the fastest time possible!

Version A - Tammy working alone, could clean one-third the area in one hour. Her partner Melissa, working alone, could clean one-third the area in one hour. During the contest when they work together, how long will it take them to clean the area? Explain how you found your solution.

Version B - Tammy working alone, could clean one -half the area in one hour. Her partner Melissa, working alone, could clean one-fourth the area in one hour. During the contest when they work together, how long will it take them to clean the area? Explain how you found your solution.

Version C - Tammy working alone, could clean one-half the area in one hour. Her partner Melissa, working alone, could clean one-third the area in one hour. During the contest when they work together, how long will it take them to clean the area? Explain how you found your solution.

Friends Meeting on Bikes - Task 6 – Mary Swanton

Taylor and Anya live 63 miles apart. Sometimes on a Saturday, they ride their bikes toward each other's houses and meet somewhere in between. Taylor is a very consistent rider - she finds that her speed is always very close to 12.5 miles per hour. Anya rides more slowly than Taylor, but she is working out and so she is becoming a faster rider as the weeks go by.

On a Saturday in July, the two friends set out on their bikes at 8 am. Taylor rides at 12.5 miles per hour, and Anya rides at 5.5 miles per hour. After one hour, how far apart are they?

a. On a separate sheet of paper, make three tables. One to show how far Taylor has travelled. One to show how far Anya has travelled. And one to show how far apart they are.

b. At what time will the two friends meet?

c. Taylor says, "If I ride at 12.5 miles per hour toward you, and you ride at 5.5 miles per hour toward me, it's the same as if you stay still and I ride at 18 miles per hour." What do you think Taylor means by this? Is she correct?

Candle Comparison
Lesson Plan

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The Task

There are two popular stores that sell expensive candles in the area. Candles R Us sells a candle that will burn for 4 hours. Candle-mania sells a candle that will burn for 5 hours. After how many hours will one candle be 3 times the size of the other?

Standards of Learning

- A.4- The student will solve multistep linear and quadratic equations in two variables, including

  d) solving multistep linear equations algebraically and graphically;
  f) solving real-world problems involving equations and systems of equations.

Facilitating Task

- Students will be able to use proportional reasoning to solve the candle comparison problem.
- I would like my students to use proportional reasoning and problem solving skills to find the solution to the candle comparison problem.
- I expect students to use pictures to represent the situation and help them develop a problem solving strategy. Also, I expect my students to use tables and graphs to try and see the pattern for the rate at which each candle is burning. I also expect my students to try and develop expressions to represent this situation and ultimately an equation that will help them to arrive at the correct solution.
- Students should use their previous problem solving skills to develop strategies to attack this problem. We have tackled many different problems in the past that required equations to solve and I expect my students to make that connection and use that skill to help them in this situation. We have looked at many rate problems and I would expect my students to understand how the rate and time are connected in order to help them.

Materials

- *Student black line masters, writing utensils, promethean board.*

Misconceptions	Suggested Prompts or Questions
• *Students need to understand that the rate that the candle is the important factor they will need in their equation or table. They may get fixated on the height of the candle and confuse the rate or 4 or 5 hours with how tall the candle is. This will result in making an equation that will not help them solve the situation.* • *Students may struggle with knowing which candle will be taller/shorter when trying to understand how the burning rate will affect the candle. It is important to clarify that the candle that burns for 5 hours will be taller because it burns more slowly in comparison to the candle that burns for 4 hours.*	• *Can we create an expression to represent the rate that the candle is burning?* • *How can we organize the data we have to see patterns?* • *How can we tell which candle is going to be taller?* • *Can we create an equation to represent this situation?* • *How can we express the whole candle vs. part of the candle as an expression?* • *What information do we know?*

Sequencing

- *When sharing ideas I started with the group that was trying to use trial and error to arrive at when the one candle would be 3 times as tall as the other one. Then, I looked at students who were arranging their ideas in tables or sequences of fractions to try to develop an expression from their thought process. Then I had a student share out an idea that had an error in it which they were able to correct with the help from their group. This led to a very strong understanding of the equation that was necessary to solve the problem. Then, I had a pair of students present that had clearly defined their variables and procedure. This helped the other students to understand the process better. Finally, I had a group present that used a formal equation and all of the steps to arrive at the solution.*

Review/Extension

- *I will make sure to coach students to organize their information in a way which they can see patterns. I will prompt them to express the candle as a whole, then how much would be left after one hour, two hours etc. Hopefully that will help them arrive at an expression that they can use to represent how much of the candle has been burned and how much of the candle is left.*
- *I plan to ask students when they think the candles will be the same size and to explain their thinking if they finish early. They will be expected to defend their reasoning. (This extension was not needed).*