

A Lewis Carroll Conundrum

A Puzzle by Lewis Carroll
Accompanying Geometer's Sketchpad activity created by Steve Klarevas
EDCI 646: School Change
Spring 2011



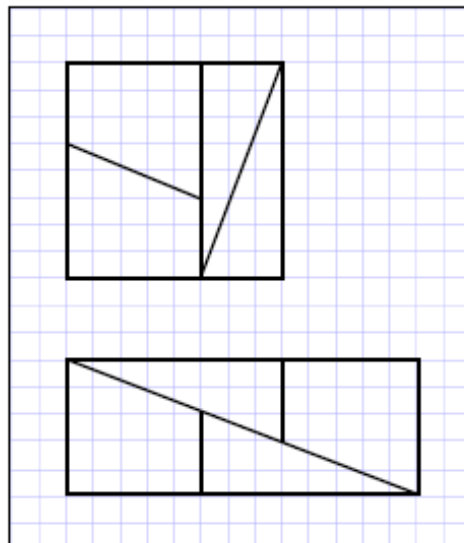
The Task

Cutting and rearranging the 8 x 8 square to the right into the 13 x 5 rectangle beneath it is a more curious task than it seems. Do you see why that is?

Compare the area of the 8 x 8 square to that of the 13 x 5 rectangle.

Hmmm, rearranging the square (area of 64 units²) into the rectangle (area of 65 units²) results in one extra square unit!

How is this possible? Can you account for the extra square unit?



Big Ideas

- Polygons and areas
- Polygon classification
- Transformations
- Slope
- Fibonacci Numbers (extension)
- Trigonometry (extension)

Standards of Learning for Grades 6-7-8

- 6.10c Solve practical problems involving area and perimeter
- 6.11b Graph ordered pairs in a coordinate plane
- 6.13 Describe and identify properties of quadrilaterals
- 7.8 Represent transformations (reflections, dilations, rotations, and translations) by graphing in the coordinate plane
- 8.8 Apply transformations to plane figures
- 8.11 Solve practical area and perimeter problems involving composite plane figures

Standards of Learning for Algebra 1 and Geometry

- A.6a Determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line
- G.3 The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include
- a) Investigating and using formulas for finding distance, midpoint, and slope;
 - b) Applying slope to verify and determine whether lines are parallel or perpendicular

Continue to next page for Process Goals and Related Tasks

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Process Goals

- Problem Solving and Reasoning – Students will make and investigate mathematical conjectures, applying and adapting a variety of appropriate strategies to solve problems that arise in mathematics.
- Connections and Representations – Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Communication – Students will justify their findings, write about them, and present their results to the class with precise mathematical language.

Related Task – How Can a Plane Just Disappear: A Geometric Probability Investigation

The task deals with the disappearance of Malaysian Airlines Flight 370. It is loosely related to “A Lewis Carroll Conundrum” as it, too, deals with ratios (geometric probability), polygons and area, and the idea and realization of “smallness.” Please see the actual task and lesson for more details.

Related Task – Harry Potter and the Age of Man

The task deals with the amount of time modern man has been on Earth as it compares to the amount of time the planet has existed, done in the context of the length of a Harry Potter novel. It is loosely related to “A Lewis Carroll Conundrum” as it, too, deals with ratios and the idea and realization of “smallness.” Please see the actual task and lesson for more details.

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Lesson Plan

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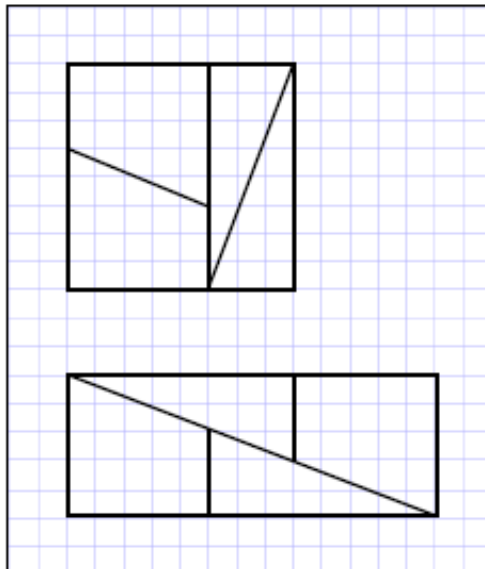
The Task

Cutting and rearranging the 8×8 square to the right into the 13×5 rectangle beneath it is a more curious task than it seems. Do you see why that is?

Compare the area of the 8×8 square to that of the 13×5 rectangle.

Hmmm, rearranging the square (area of 64 units²) into the rectangle (area of 65 units²) results in one extra square unit!

How is this possible? Can you account for the extra square unit?



Materials

- Handouts (the task itself and the Geometer's Sketchpad activity, blackline masters 1 and 2, respectively)
- Computers with Geometer's Sketchpad software
- Paper
- Pencils
- Scissors
- Glue or tape
- Calculators

Facilitating Task

- Task may be done individually or in pairs.
- Read the task together and answer clarifying questions.
- Students will be given any ancillary materials upon request.
- Give students individual think time prior to task execution.
- Students present findings during last 10-15 minutes of class.

Misconceptions

- That the pieces fit together seamlessly in the rectangle. That is not the case. (The rectangle's diagonal is drawn so thickly as to obscure a gap.)
- This may be a trick, but it is not just a trick. The task is rich in mathematical content.

Suggested Prompts or Questions

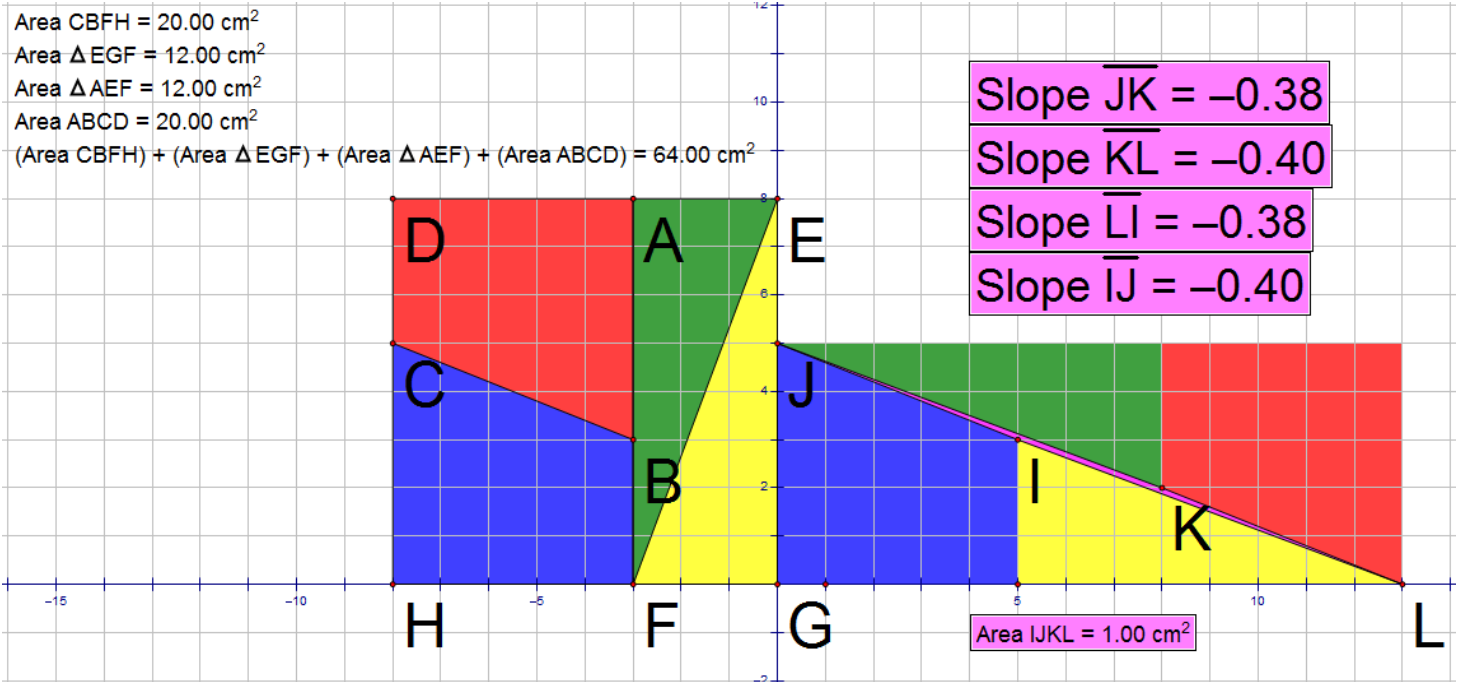
- Where do you think the extra area is coming from? What do you think is happening?
- Think about slopes.
- Extension: The various slopes are $-2/5$, $-5/13$, and $-3/8$. Do you recognize anything special about 2, 3, 5, 8, and 13? (They are Fibonacci numbers.)

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Name KEY

Date _____



Notice that when rearranged the sides of the triangles and trapezoids that seemingly come together to form the rectangle's diagonal in fact do not. They are slightly mismatched owing to subtly different slopes, $-2/5$ versus $-3/8$. The result of this mismatch is a gap in the rectangle in the form of a sliver of a parallelogram, here named IJKL, with an area of—you guessed it—1 unit². IJKL is a parallelogram since both pairs of opposite sides are parallel.

A Lewis Carroll Conundrum



Name _____

Date _____

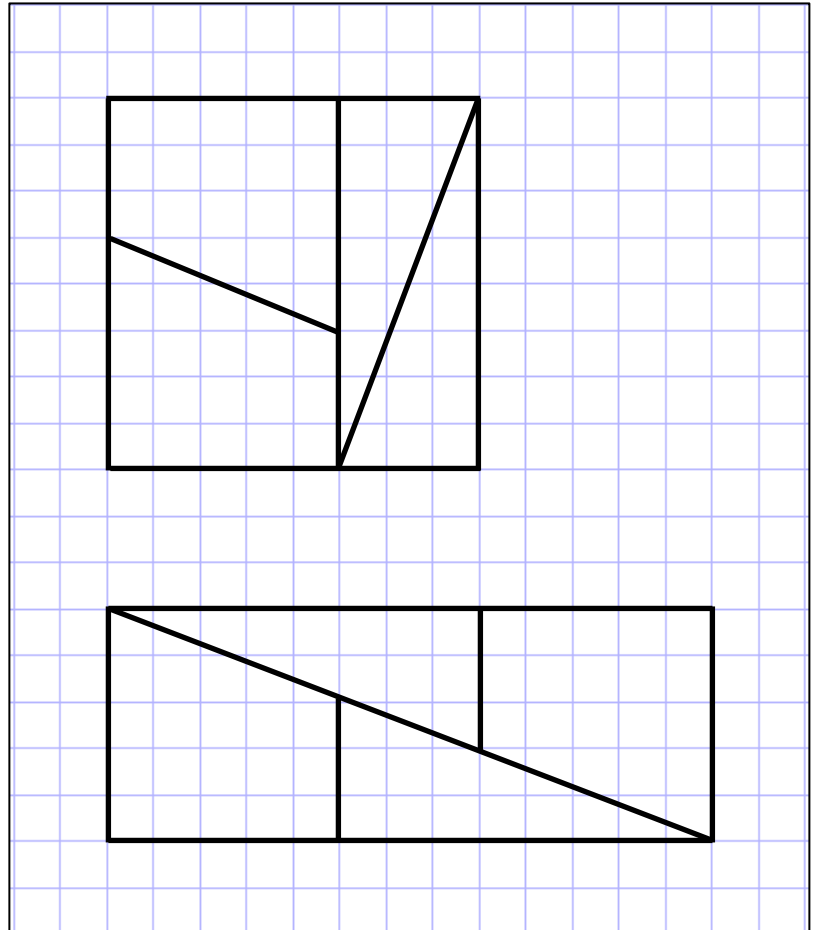
Cutting and rearranging the 8×8 square to the right into the 13×5 rectangle beneath it is a more curious task than it seems. Do you see why that is?

Compare the area of the 8×8 square to that of the 13×5 rectangle.

Hmmm, rearranging the square (area of 64 units^2) into the rectangle (area of 65 units^2) results in one extra square unit!

How is this possible? Can you account for the extra square unit?

The next page has a larger square for you to dissect.

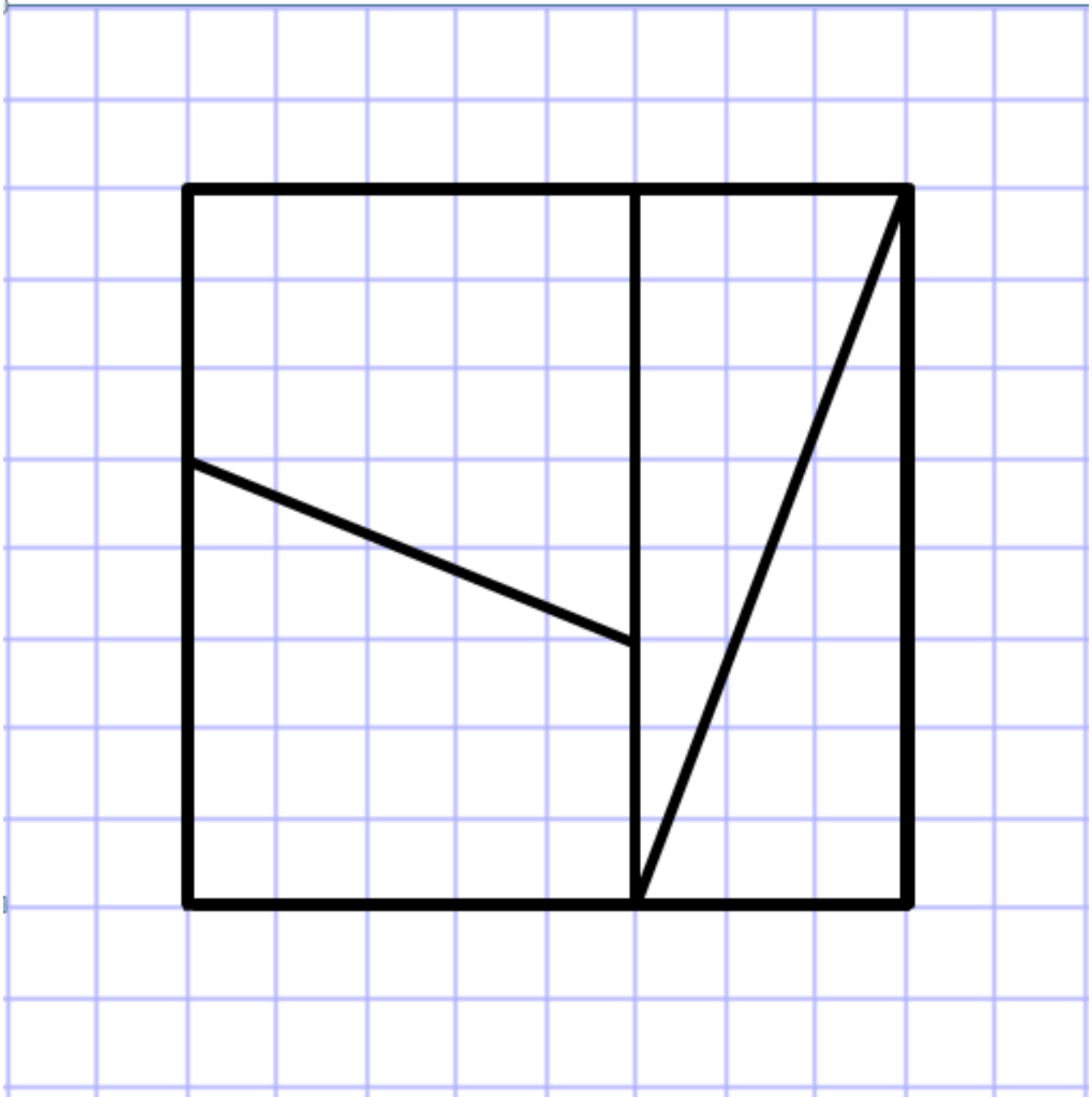


Lewis Carroll Conundrum



Name _____

Date _____



Lewis Carroll Conundrum Geometer's Sketchpad Worksheet



1. Select **Graph > Show Grid**.
2. Select **Graph > Snap Points**.

You will construct the square in Quadrant II with two of its sides along the x- and y-axes.

3. Select the **Point Tool** and place points at (0, 8) and (-3, 0).
4. Using the **Selection Arrow Tool**, highlight the points at (0, 0), (0, 8), and (-3, 0).
5. Select **Construct > Triangle Interior**.

Classify the triangle by both its angles and sides: _____

6. With the triangle interior highlighted, right click within it and select **Color > choose a color**.
7. (Left) Click in any empty space outside of the triangle. Highlight the three vertices once more and select **Construct > Segments**.

8. Select the **Point Tool** and place points at (-8, 0), (-8, 5), and (-3, 3).
9. Using the **Selection Arrow Tool**, highlight the points at (-8, 0), (-8, 5), (-3, 3), and (-3, 0) either in clockwise or counterclockwise order.
10. Select **Construct > Quadrilateral Interior**.

What type of quadrilateral is this specifically? _____

11. With the quadrilateral interior highlighted, right click within it and select **Color > choose a different color from before**.
12. (Left) Click in any empty space. Highlight the four vertices once more—either in clockwise or counterclockwise order—and select **Construct > Segments**.

Repeat this process to complete the entire square. Choose a different color for each polygon within the square.

13. Click in any empty space. Highlight all polygon interiors by clicking within each one. Select **Measure > Area**. The four areas will be displayed.
14. Select **Measure > Calculate**. Click each area and add them. Press **OK**.

What is the area of the square? _____

Besides adding the areas of the polygons to determine the area of the square, how else could you have calculated the area of the square? _____

Next, you will rearrange the pieces to form the rectangle in Quadrant I with two of its sides along the x- and y-axes.

15. Highlight the interior of the bottom quadrilateral that is resting on the x-axis. Select **Transform > Translate**. Choose *rectangular translation vector*. Enter 8 for the horizontal component and 0 for the vertical component. (*Why?*) Press **Translate**.

Lewis Carroll Conundrum Geometer's Sketchpad Worksheet

16. Highlight the interior of the triangle that has a vertex at the Origin. Double click the Origin point. Select **Transform > Rotate**. Try entering a few angles, both positive and negative, but do *not* press **Rotate** yet. Look at the silhouettes of the triangle.

Describe the direction a polygon moves in when it is rotated positive degrees about a point of rotation (in this activity the point about which the triangle is rotated is the Origin). _____

Describe the direction a polygon moves in when it is rotated negative degrees about a point of rotation. _____

Enter -90 degrees and press **Rotate**.

17. With the new (image) triangle highlighted, select **Transform > Translate**. Choose *rectangular translation vector*. Enter 5 for the horizontal component and 0 for the vertical component. Press **Translate**.
18. Click in any empty space. Highlight the intermediate step triangle, the one that resulted from the -90 degree rotation, but before the translation. Be careful not to select the quadrilateral. Select **Display > Hide Triangle**.

Repeat this process to complete the entire rectangle. Carefully hide any unnecessary shapes cluttering your sketch.

The dimensions of the rectangle are _____ units x _____ units, and the area of the rectangle is _____ units².

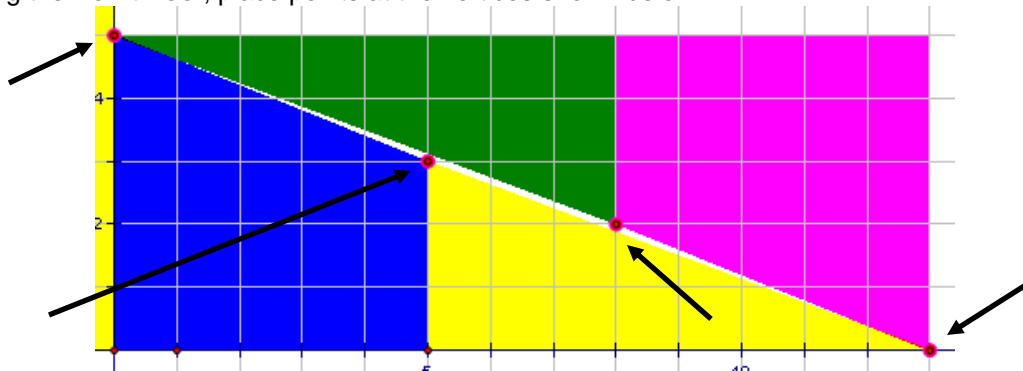
Examine the rectangle carefully. Do you notice anything unusual? Explain in detail why there is a discrepancy between the area of the square and the area of the rectangle. _____



Do not proceed to page 3 until pages 1 and 2 have been *completely finished*.

Lewis Carroll Conundrum Geometer's Sketchpad Worksheet

19. Using the **Point Tool**, place points at the vertices shown below.



20. Using the **Selection Arrow Tool**, highlight the four points either in clockwise or counterclockwise order.

21. Select **Construct > Quadrilateral Interior**.

22. With the quadrilateral interior highlighted, right click within it and select **Color > choose yet a different color**.

23. With the quadrilateral interior still highlighted, select **Measure > Area**.

Is this the area measurement that you expected? Explain. _____

24. Click in any empty space. Highlight the four vertices once more—either in clockwise or counterclockwise order—and select **Construct > Segments**.

25. With the segments still highlighted, select **Measure > Slope**.

What are the slopes of the four sides of this “tiny sliver” quadrilateral?

Now that you know the slopes, what type of specific quadrilateral is this “tiny sliver”? _____

How does this slope information fit in with and support your explanation at the bottom of page 2? _____
