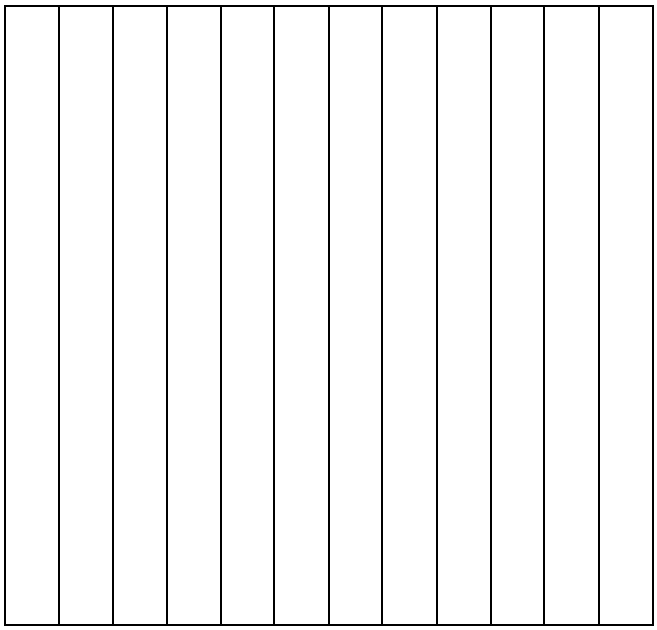
Brownie Pan

Designed by:
Jonathan Thompson
George Mason University, COMPLETE Math

The Task

Mr. Brown E. Pan recently opened a new business making brownies called "The Brown E. Pan." On his first day baking, he started in his own kitchen by using a single square pan. He just finished his first batch of brownies, and he wants to make sure that all the brownies are the same size. He first cut the brownies vertically as shown in the picture below.



The pan of brownies above is cut into 12 congruent rectangles. If the perimeter of each of the rectangles is 65 cm., what is the area of the pan?

Big Ideas

- Using perimeter, area, and volume
- Finding relationships between perimeter, area, and volume
- Measurement

Standards of Learning for Grades 3-4-5

- 3.9 – The student will estimate and use U.S. Customary and Metric Units to measure:
  - Length
  - Liquid Volume
  - Weight/Mass
  - Area and Perimeter
- 4.6 - The student will
  - Estimate and measure weight/mass and describe the results in U.S. Customary and Metric Units
  - Identify equivalent measurments between units
- 4.7 - The student will

Standards of Learning for Grades 6-7-8

- 6.9 – The student will make ballpark comparisons between measurements in the U.S. Customary system and the metric system.
- 6.10 – The student will
  - Solve practical problems involving area and perimeter
  - Describe and dermine the volume of a rectangular prism
- 7.5 – The student will
  - Describe volume of cylinders
  - Solve practical problems involving the volume of rectangular prisms and

- estimate and measure length and describe the result in both Metric and U.S. Customary
- Identify equivalent measurments between units

- 5.8 – The student will
  - Find perimeter, area, and volume in standard units of measure
  - Differentiate between perimeter, area, and volume
  - Identify equivalent measurments within the metric system
  - Choose an appropriate unit of measure for a given situation

cylinders
- Describe how changing one measured attribute of a rectangular prism affects its volume

- 8.7 – The student will
  - Investigate and solve practical problems involving volume of prisms and cylinders
  - Describe how changing one measured attribute of a figure affects the volume and surface area

Process Goals

- Problem Solving and Reasoning – Students will apply their knowledge of area and perimeter to find the area of a pan of brownies that is broken down into 12 congruent rectangles, after being cut vertically, given the perimeter of one of these 12 rectangles.
- Connections and Representations – Students will relate the perimeter of the smaller rectangles to the dimensions of the larger square. The students will do this by using reasoning skills and by showing their thinking in words and in pictures.
- Communication – Students will use mathematical language to justify their finding snd discuss solution pathways with their peers.

Related Task – Brownie Batter

Mr. Brown E. Pan recently opened a new business making brownies called "The Brown E. Pan." On his first day baking, he started in his own kitchen by using a single rectangular pan. He has already prepared 10 mixing bowls full of brownie batter.
Use pictures, words, tables, graphs, and/or symbols to figure out a plan showing how many brownies he can make.

Related Task – Bite Size Brownies

Mr. Brown E. Pan recently opened a new business making brownies called "The Brown E. Pan." On his first day baking, he started in his own kitchen by using a rectangular pan.

1. Draw a picture to show how many brownies would fill the pan.
2. Since he wanted to make the most of his time, he wondered, "What if I make the brownies smaller and add another row of brownies?" How many brownies would now fit into the pan? What if he added another row?
3. Continue the pattern to find the x stage. Find a way to record your results.
4. If the pan held 120 brownies, how many times did Mr. Brown E. Pan add a new row?
5. If he continues this pattern, would it be possible for a pan to hold 500 brownies? Explain how you know.

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Lesson Plan

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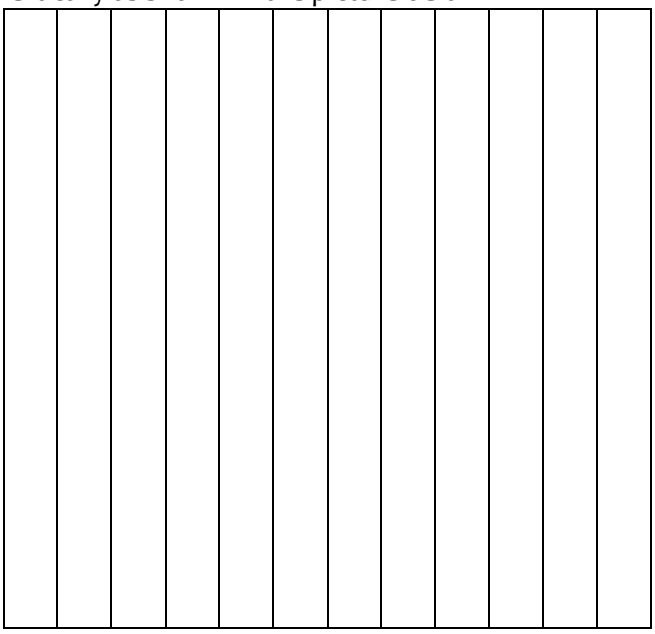

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Materials

- Task sheet for each student
- Task to project on board
- Strips of rectangular paper to visualize pan
- Graph paper
- Rulers
- Presentation paper for each group

Facilitating Task

- Class will be distributed the problem sheet which will be read together and clarifying questions will be answered.
- Student will be split into groups of 3-4 students.
- Before groups work together, each student will be given 5-7 minutes of independent time to begin working on ideas of their own.
- Independent work will transition to small group work (within the 3-4 person groups)
- Groups will be distributed materials when requested
- Groups present findings based upon strategies chosen in order by teacher.

Misconceptions

- Misinterpretation of the picture mistakenly thinking it is drawn exactly to scale
- Understanding that the pan is a square

Suggested Prompts or Questions

Questions to Engage Students That are Stuck

- Could you draw or label the information that you are given? How could you visualize what the problem is saying?
- Do you think this picture is the exact

• Confusion between properties of a square vs. properties of a rectangle • Using 65 centimeters as the area of each rectangle rather than the perimeter • Misunderstanding of the question that is being asked. Finding only the length of each side of the square, or finding the perimeter of the square instead of the area.	same size as the actual pan? • What information are we attempting to find? • What do you know about squares or rectangles? • What would happen when the widths of the rectangles are put together? • Would any rectangle work as the pan length, or does it have to be a square? • How does a square differ from a rectangle?

Brownie Pan

Name_____
Date_____

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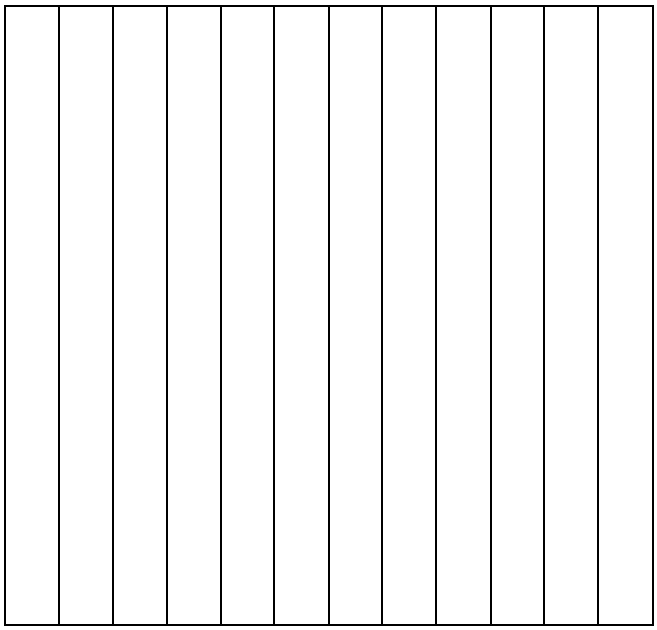

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Brownie Pan
Anticipation Guide

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George Mason University, COMPLETE Math

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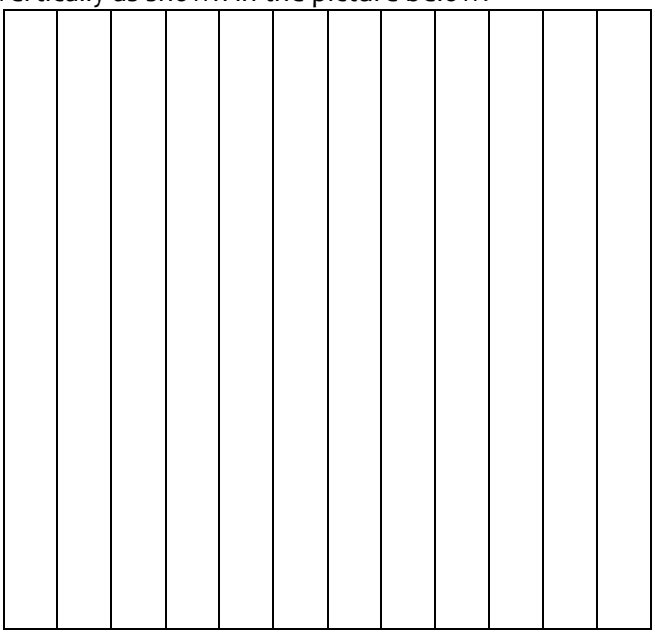

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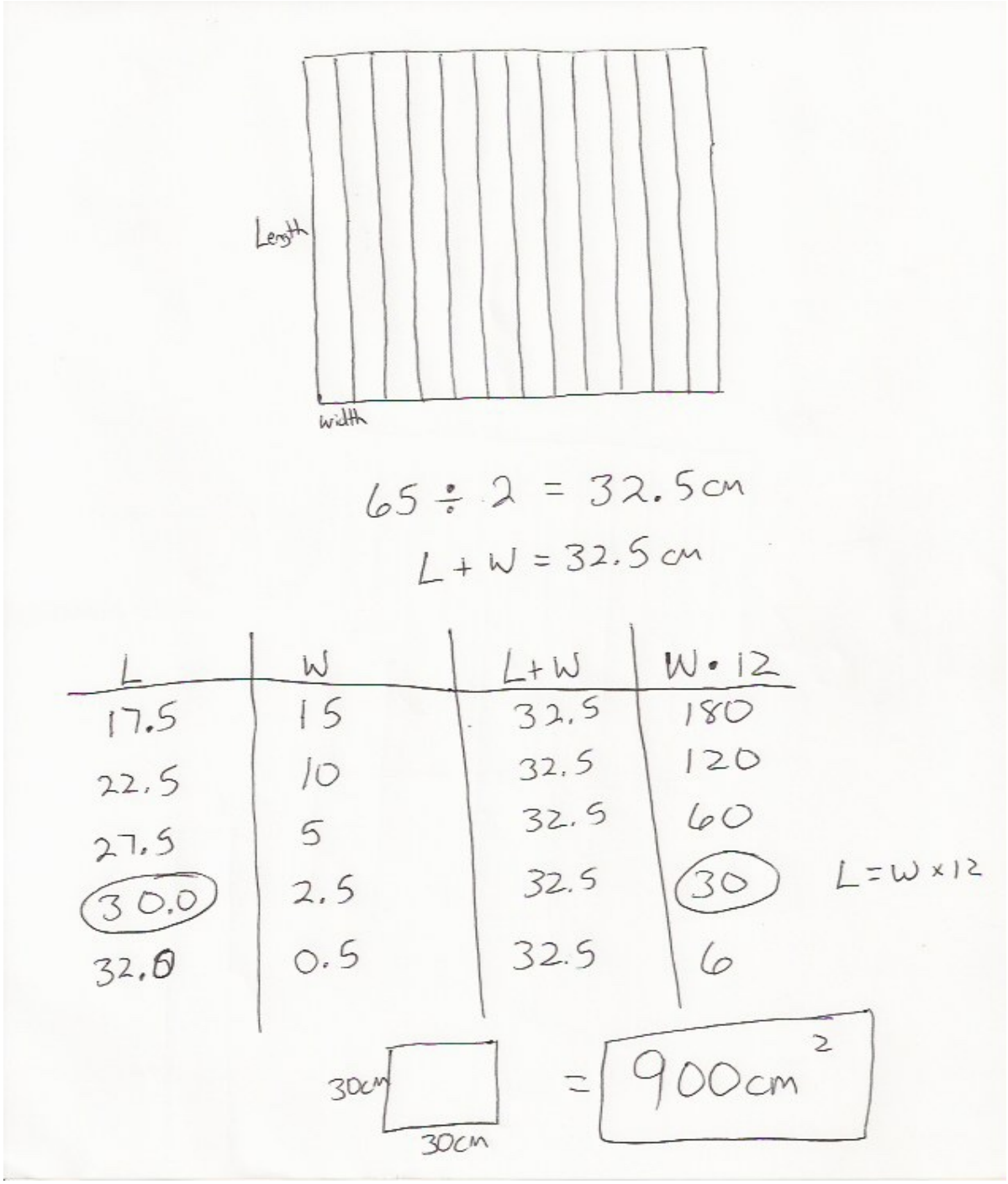
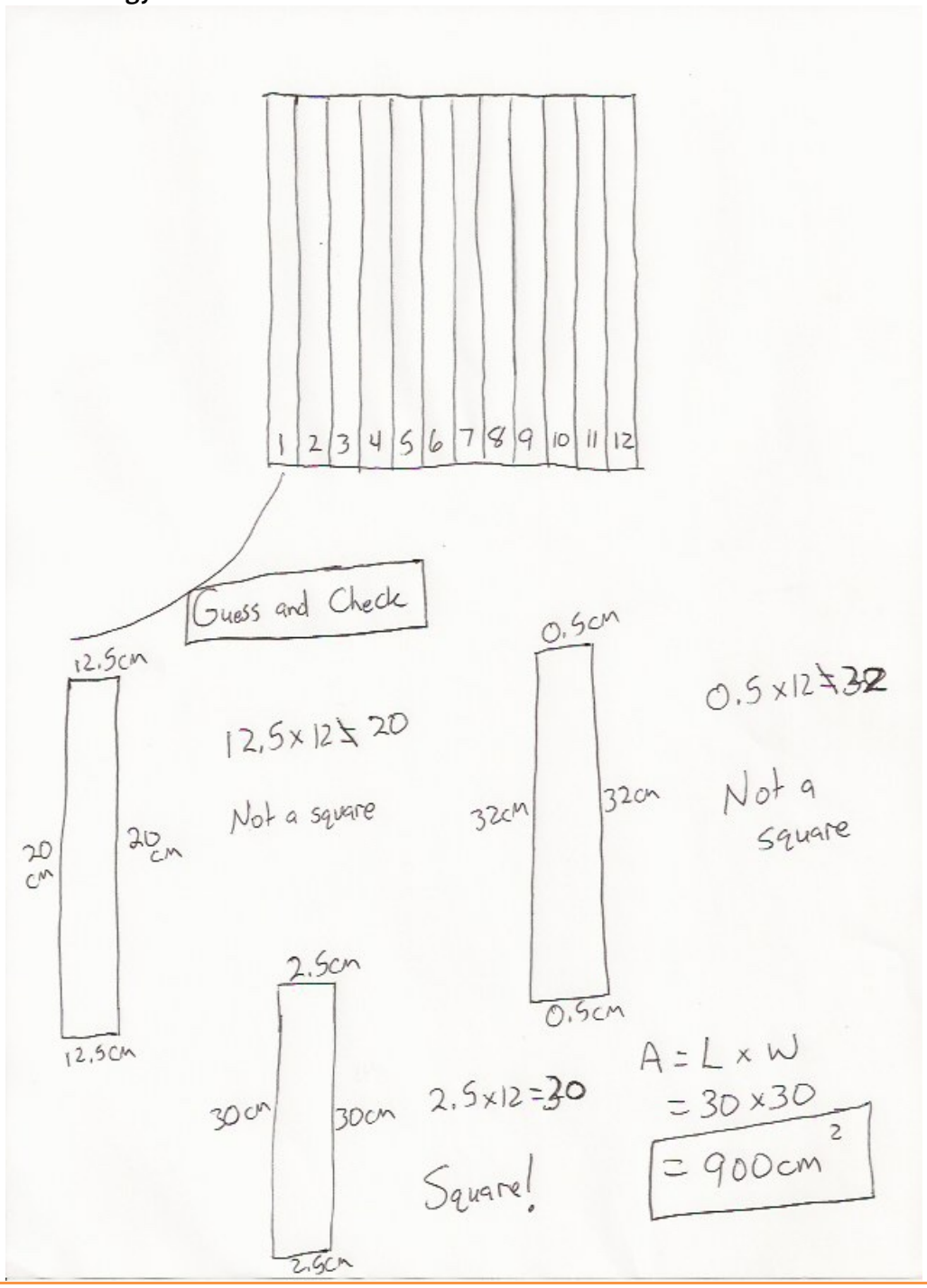
Anticipated Strategy #1



L	W	L+W	W • 12
17.5	15	32.5	180
22.5	10	32.5	120
27.5	5	32.5	60
30.0	2.5	32.5	30
32.0	0.5	32.5	6

Description:
Students may find a solution by using a T-chart. They could do this by first figuring out that the sum of the length and width of each rectangle must be half of the perimeter, or 32.5 centimeters. Upon finding this, they could make a chart with many different possible combinations of values that could add together to make 32.5 centimeters. They could find some combinations in the chart, until they come upon that works in the problem. The last column in the chart would be the check to see if it works. Since there are 12 congruent rectangles, the sum of the widths of the 12 rectangles must be equivalent to the length of the rectangle because the figure is stated as a square. The students would need to continue to do this until the length matches the product of the width and 12. This works for 30 centimeters, so the area of the entire square is 900 square centimeters.

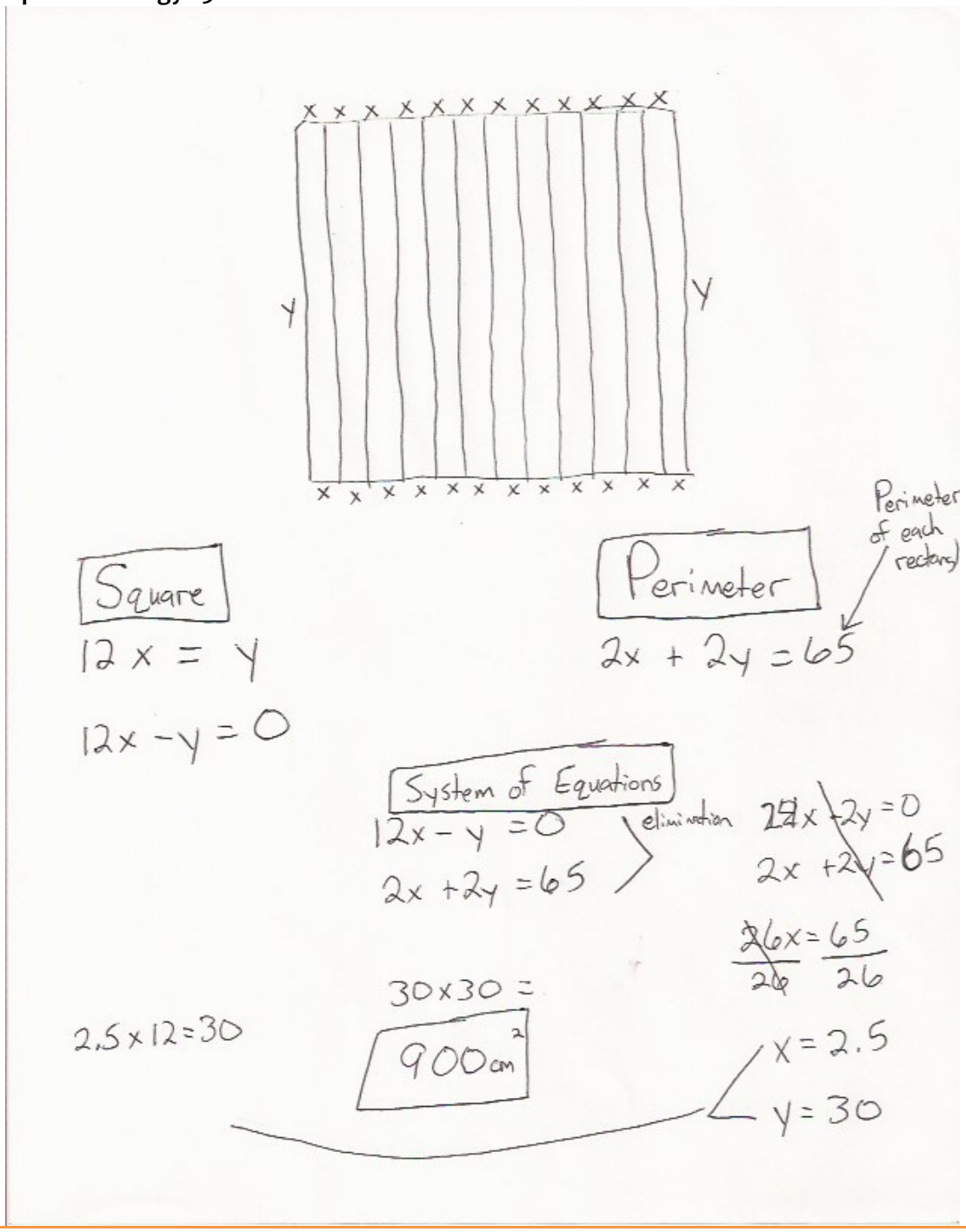
Anticipated Strategy #2



Description:

The students may solve the problem by using a pure guess and check strategy. Since the students know that each rectangle has a perimeter of 65 centimeters, this is the information that they must start with. Upon deciding this, they will most likely decide that they need to know the dimensions of these rectangles, but they will determine that there are different possible combinations. The students could find different combinations until they find one that makes a square. Since the 12 widths of the thin rectangles are equivalent to one of the lengths of the rectangles, they could continue this pattern until multiplying the width by 12 is the same as the length. This would prove the dimensions would make a square, which is stated in the problem. Once the student finds this, they can multiply the length by the width, which should be the same number, to find the area. In this case, the area is 900 square centimeters.

Anticipated Strategy #3



Description:

Students that are familiar with variables or setting up equations may look at this problem in a completely different way. First of all, they know that the shape is a square, so they may look at the two sides of the square as being equivalent. They could do this by assigning a variable (w) to the width of the thin rectangles and the variable (y) for the length of these rectangles. They could then conclude that 12x = y, since there are 12 of these smaller rectangles. They might then figure out that 2x + 2y = 65, since 65 centimeters was given as the perimeter for each of these smaller rectangles. By having these two equations, the student has created a system that could be solved in multiple ways. One way is by using elimination, which is shown above. Once the student has figured out the y-value, they could multiply this number by itself because the shape was identified as a square. This would give an area of 900 square centimeters.

Brownie Pan
Student work

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George Mason University, COMPLETE Math
Fall 2016

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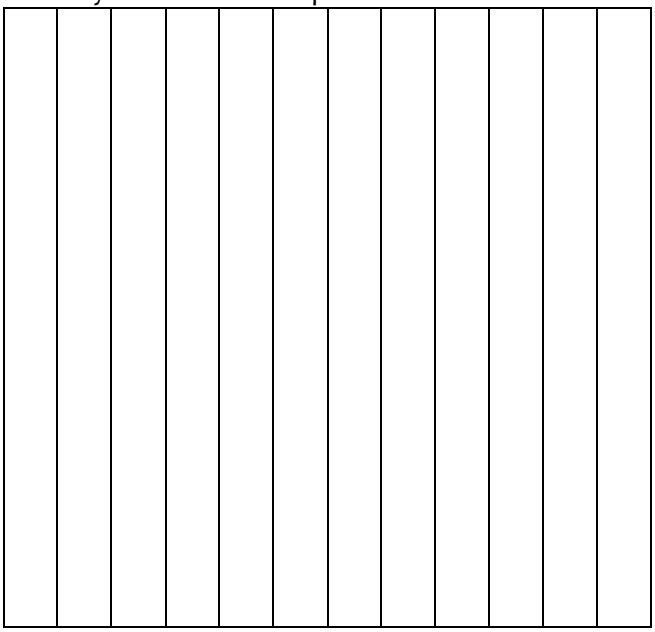

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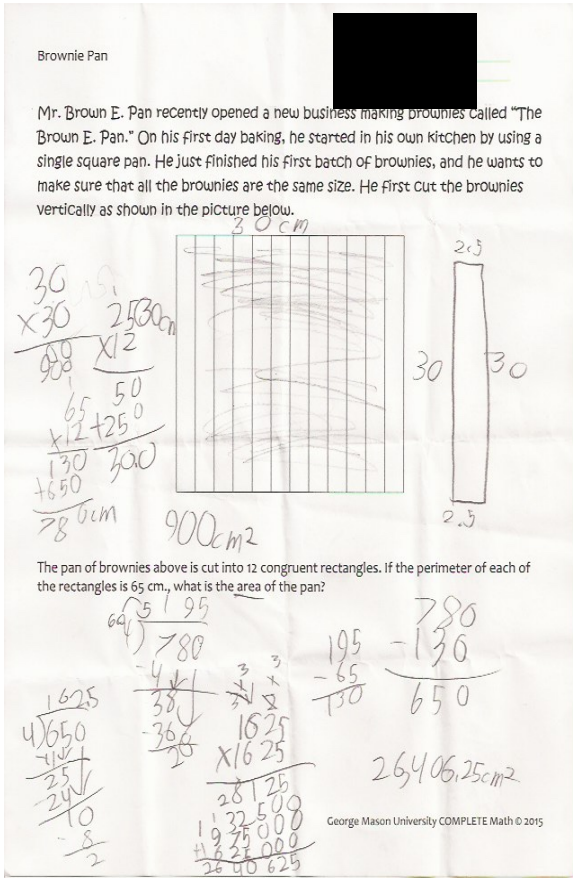
Student work 1



Brownie Pan

[redacted]

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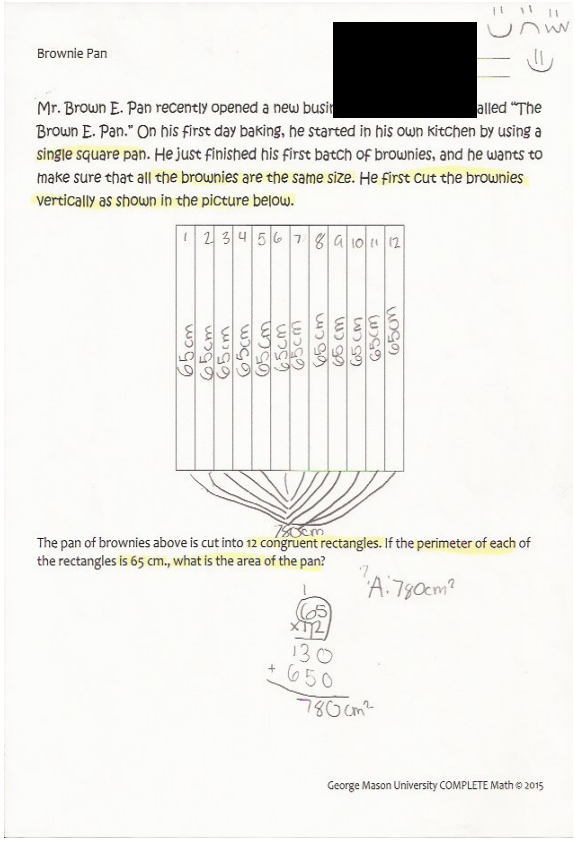
George Mason University COMPLETE Math © 2015

Teacher Notes:

This student looks to have started out with the number 65 centimeters, since this number was given as the perimeter of each of the 12 rectangles on the inside of the square. The student seems to have started by multiplying this number by 12, mistaking it as the area of each of these rectangles. After doing this, the student drew one of the smaller rectangles on the inside, which is drawn on the right of the page, and the student labeled the sides to make the perimeter of 65 centimeters. The student checked to make sure this was true by multiplying 2.5 by 12 to prove the width x 12 was equivalent to the length. This proved the shape was a square with those dimensions. The student finished by multiplying 30 by 30 to get a solution of 900 square centimeters.

Student work 2



Brownie Pan

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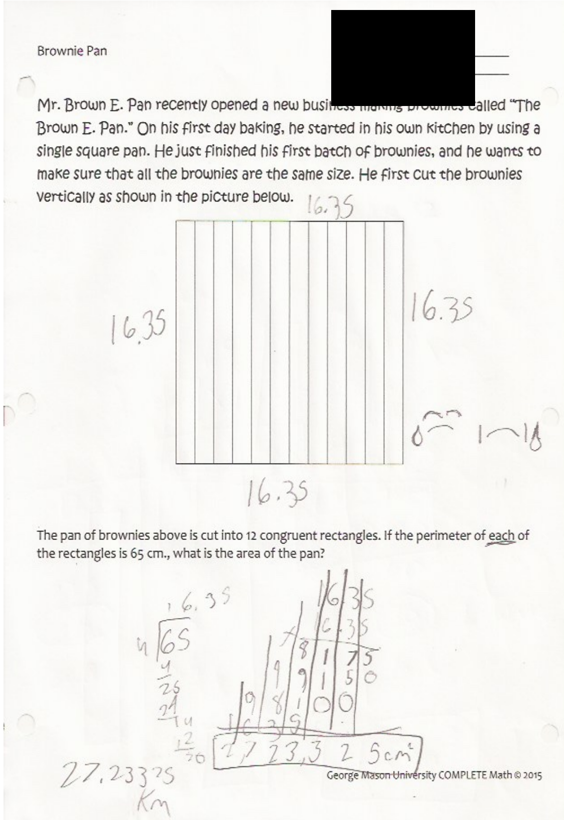
Teacher Notes:

This piece of student work illustrates what is probably the most common misconception that occurs when students work through this problem. The student started off by highlighting important information, marking things like square pan, congruent rectangles, perimeter of 65 centimeters, and the question being asked. After doing this, the student mistakenly labeled every single rectangle inside the square as 65 centimeters. While the perimeter of each of these rectangles is 65 centimeters, this student labeled this value as the area of each of these rectangles. Once the student mistakes this piece of information for the area, the problem seems extremely easy. If you know the area of one rectangle, then all you must do is multiply this value by 12 (since there are 12 equivalent rectangles) to get the total area of the whole shape. Even though this student highlighted the word perimeter, the setup of the problem and this one mistake led to a complete misconception.

Student work 3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Teacher Notes:

This student started out this problem with the main piece of information given, the perimeter of the smaller rectangles that fill the larger square, which is given as 65 centimeters. Once the student saw this number along with the perimeter, the student chose to divide this number by 4. This is a strategy that makes some sense because each of these rectangles have 4 sides, so dividing by 4 is a logical way to get the value of each side. This would work in the case of the rectangle being a square, but in this picture, these rectangles are clearly not. Once doing this, the student labeled each side of the larger square as 16.35, which was intended to be 16.25 if 65 was correctly divided by 4. Once the student had mistakenly labeled the square with these side lengths after misinterpreting the information, the student multiplied the length by the width (these values were the same) to get the final solution to the problem. This is a common mistake in this problem, but it is one that can be prevented when closely reading the information given.

Sequencing

- At least four groups will present their findings for determining the area of the square baking pan.
  - The first group will show a strategy where they visualized the problem using either rectangular paper strips or graph paper. They might not have actually found the length and width of the rectangles, but they will discuss the meaning of the given information. They will discuss how two numbers need to add to get the perimeter of each of the squares. They will discuss how the sides of the rectangles compare to the larger square.
  - The second group will display a guess-and-check method. They will discuss how they looked at the perimeter of the rectangles and guessed and checked their work to make sure the perimeter also worked to make a larger square. They will show how they could prove that this could be the only possible set of numbers to work.
  - The third group will be one that works backwards through the problem. They will start with the square and set the side lengths of the square as a given number and to see if this works for their problem. They will show how you could also work backwards to find the solution.
  - The fourth group will show a little more sophisticated strategy if one exists, such as using variables to stand for the unknowns in the problem or a different creative strategy. They will discuss how their strategy relates to the others.