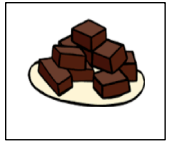


Brownie Batter



Designed by:
Jonathan Thompson
George Mason University, COMPLETE Math

The Task

Mr. Brown E. Pan recently opened a new business making brownies called “The Brown E. Pan.” On his first day baking, he started in his own kitchen by using a single rectangular pan. He has already prepared 10 mixing bowls full of brownie batter.

Use pictures, words, tables, graphs, and/or symbols to figure out a plan showing how many brownies he can make.

Big Ideas

- Using perimeter, area, and volume
- Finding relationships between perimeter, area, and volume
- Measurement

Standards of Learning for Grades 3-4-5

- 3.9 – The student will estimate and use U.S. Customary and Metric Units to measure:
 - Length
 - Liquid Volume
 - Weight/Mass
 - Area and Perimeter
- 4.6 - The student will
 - Estimate and measure weight/mass and describe the results in U.S. Customary and Metric Units
 - Identify equivalent measurements between units
- 4.7 - The student will
 - estimate and measure length and describe the result in both Metric and U.S. Customary
 - Identify equivalent measurements between units
- 5.8 – The student will
 - Find perimeter, area, and volume in standard units of measure
 - Differentiate between perimeter, area, and volume
 - Identify equivalent measurements within the metric system
 - Choose an appropriate unit of measure for a given situation

Standards of Learning for Grades 6-7-8

- 6.9 – The student will make ballpark comparisons between measurements in the U.S. Customary system and the metric system.
- 6.10 – The student will
 - Solve practical problems involving area and perimeter
 - Describe and determine the volume of a rectangular prism
- 7.5 – The student will
 - Describe volume of cylinders
 - Solve practical problems involving the volume of rectangular prisms and cylinders
 - Describe how changing one measured attribute of a rectangular prism affects its volume
- 8.7 – The student will
 - Investigate and solve practical problems involving volume of prisms and cylinders
 - Describe how changing one measured attribute of a figure affects the volume and surface area

Process Goals

- Problem Solving and Reasoning – Students will use mathematical modeling to reason through how many pans of brownies (and an actual number of brownies) could be made given 10 mixing bowls of brownie batter.
- Connections and Representations – Students will use various tools to gather information on how many pans of brownie could be made from a gallon of brownie batter, and they will generalize their thinking to figure out how many brownies could be made from 10 mixing bowls of brownie batter. They will represent their thinking in pictures, tables, and in the actual modeling itself.
- Communication – Students will use mathematical language to justify their finding and discuss solution pathways with their peers.

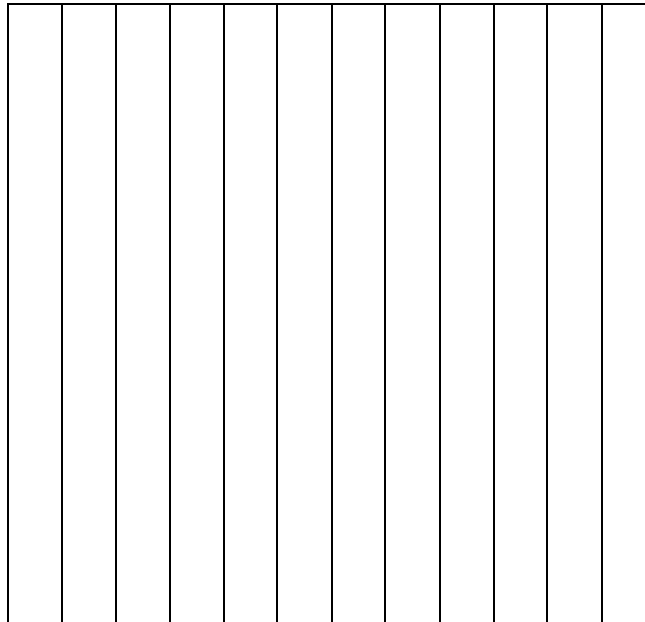
Related Task – Bite Size Brownies

Mr. Brown E. Pan recently opened a new business making brownies called “The Brown E. Pan.” On his first day baking, he started in his own kitchen by using a rectangular pan.

1. Draw a picture to show how many brownies would fill the pan.
2. Since he wanted to make the most of his time, he wondered, “What if I make the brownies smaller and add another row of brownies?” How many brownies would now fit into the pan? What if he added another row?
3. Continue the pattern to find the x stage. Find a way to record your results.
4. If the pan held 120 brownies, how many times did Mr. Brown E. Pan add a new row?
5. If he continues this pattern, would it be possible for a pan to hold 500 brownies? Explain how you know.

Related Task – Brownie Pan

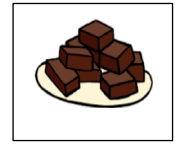
Mr. Brown E. Pan recently opened a new business making brownies called “The Brown E. Pan.” On his first day baking, he started in his own kitchen by using a single square pan. He just finished his first batch of brownies, and he wants to make sure that all the brownies are the same size. He first cut the brownies vertically as shown in the picture below.



The pan of brownies above is cut into 12 congruent rectangles. If the perimeter of each of the rectangles is 65 cm., what is the area of the pan?

Brownie Batter

Lesson Plan



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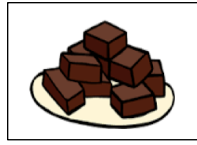
The Task

Mr. Brown E. Pan recently opened a new business making brownies called “The Brown E. Pan.” On his first day baking, he started in his own kitchen by using a single rectangular pan. He has already prepared 10 mixing bowls full of brownie batter.

Use pictures, words, tables, graphs, and/or symbols to figure out a plan showing how many brownies he can make.

<p style="text-align: center;">Materials</p> <ul style="list-style-type: none"> • Task sheet for each student • Task to project on board • Different size rectangular baking pans • Different size mixing bowls • Graduated Cylinders • Water (used as brownie batter substitute) • Graph paper • Rulers • Presentation paper for each group 	<p style="text-align: center;">Facilitating Task</p> <ul style="list-style-type: none"> • Class will be distributed the problem sheet which will be read together and clarifying questions will be answered. • Student will be split into groups of 3-4 students. • Before groups work together, each student will be given 5-7 minutes of independent time to begin working on ideas of their own. • Independent work will transition to small group work (within the 3-4 person groups) • Groups will be distributed materials when requested • Groups present findings based upon strategies chosen in order by teacher.
<p style="text-align: center;">Misconceptions</p> <ul style="list-style-type: none"> • There is not one solution to the problem • Determining the relationships between different type of capacity measurements both standard and metric • A single mixing bowl does not necessarily have to make a whole number of pans of brownies • Brownies should be the same size • Relationship between linear volume ($l \times w \times h$) to capacity measurements. 	<p style="text-align: center;">Suggested Prompts or Questions</p> <ul style="list-style-type: none"> • How could you use the tools that are available to you? • How could you figure out the volume that would be needed for a pan of brownies? Could you measure with a ruler? How could you use this? • How could you split up a single mixing bowl? How many pans of brownies would that make? • How big should the brownies be? Why would this matter? Would it matter if they are the same size or not? • How can we measure the pan to determine the volume of batter it would need? If we get a volume in cubic units, how does this compare to gallons or other capacity measurements?

Brownie Batter



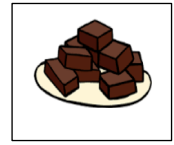
Name _____

Date _____

Mr. Brown E. Pan recently opened a new business making brownies called “The Brown E. Pan.” On his first day baking, he started in his own kitchen by using a single rectangular pan. He has already prepared 10 mixing bowls full of brownie batter.

Use pictures, words, tables, graphs, and/or symbols to figure out a plan showing how many brownies he can make.

Brownie Batter Anticipation Guide



Designed by:
Jonathan Thompson
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The Task

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Use pictures, words, tables, graphs, and/or symbols to figure out a plan showing how many brownies he can make.

Anticipated Strategy #1

Mixing Bowl
Capacity of 3 quarts of brownie batter
 $\times 10 \text{ bowls} = 30 \text{ total quarts of batter}$

$L = 13 \text{ in}$
 $H = 1.5 \text{ in}$
 $W = 9 \text{ in}$

$V = L \times W \times H$
 $= 13 \times 9 \times 1.5$
 $= 175.5 \text{ in}^3 \text{ for each pan}$

$1 \text{ in}^3 \approx 1 \text{ tablespoon}$
 $1 \text{ quart} = 64 \text{ tablespoons}$
 $64 \text{ in}^3 \approx 1 \text{ quart}$

$30 \div 2.7 = 11 \text{ batches/pans of brownies}$

$175.5 \div 64 \approx 2.7 \text{ quarts per pan}$

$11 \times 12 \text{ brownies per pan}$
 $= 132 \text{ brownies}$

Description:

Students might solve the problem by first starting off with some prior knowledge of the size of a mixing bowl. Based upon previous baking or measuring experience, they may know a relative estimated size of how big the bowl could be. This could also come from mixing bowls that are made available to the students. By multiplying this number by 10, they would have the total amount of batter. After they have this, they could estimate or measure the size of the pan and convert to figure out how much batter they would need for one pan. To finish, they would divide the total amount of batter by the amount needed per pan to get the total number of pans that could be baked. From this point, they would only need to figure out how many brownies are in each pan to get a total number.

Anticipated Strategy #2

Each mixing bowl = 1 pan of brownies

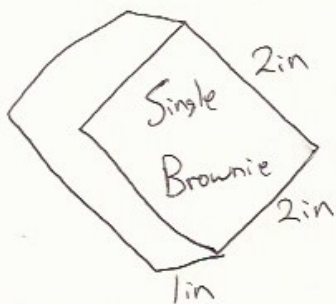
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24

$10 \text{ pans} \times \frac{24 \text{ brownies}}{\text{pan}} = 240 \text{ brownies}$

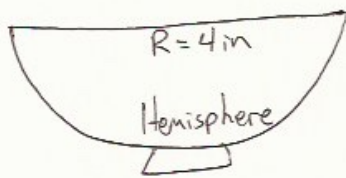
Description:

Students may relate the mixing bowls to the pans in a 1 to 1 ratio. Through previous baking experience, they are most likely used to using a mixing bowl to make only one pan of brownies. Due to this, they might think that each mixing bowl is restricted to creating one pan. If this is true, then 10 batches of brownies are possible. The student would then have to figure out how many brownies could fit into one single pan. This could be an estimate based upon a picture, or they could actually use a real pan and measure to get a more accurate estimate. Upon knowing this, they could multiply the amount of brownies in each pan by the total number of pans to get the total number of brownies.

Anticipated Strategy #3



$$\begin{aligned}V &= L \times W \times h \\ &= 2 \times 2 \times 1 \\ &= 4 \text{ in}^3 \text{ per brownie}\end{aligned}$$



$$\begin{aligned}V &= \left(\frac{4}{3} \pi r^3\right) \div 2 \\ &= \left(\frac{4}{3} \pi 4^3\right) \div 2 \\ &\approx 134 \text{ in}^3 \\ &\quad \times 10 \text{ bowls} \\ &\hline &1340 \text{ in}^3\end{aligned}$$

$$1340 \text{ in}^3 \div 4 \text{ in}^3 \approx \boxed{335 \text{ brownies}}$$

Description:

Students might start off by thinking about the size of one single brownie. This could be drawn on a piece of paper or cut out to place inside a pan to make sure of the size. Once this size is determined, they could find the volume of each brownie that would be baked. After doing this, they could think about the volume of a pan. They could do this by estimating, by using prior knowledge, or they could look at the pan as half of a sphere and find the volume. This number would need to be multiplied by 10 to accommodate each mixing bowl. The total volume of all 10 mixing bowls would be divided by the volume of each brownie to arrive at a total number of brownies. This answer could vary widely based upon how big the students think a brownie could be.

Brownie Batter

Student work

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Use pictures, words, tables, graphs, and/or symbols to figure out a plan showing how many brownies he can make.

Student work 1

Brownie Batter



Mr. Brown E. Pan recently opened a new business making brownies called "The Brown E. Pan." On his first day baking, he started in his own kitchen by using a single rectangular pan. He has already prepared 10 mixing bowls full of brownie batter.

Use pictures, words, tables, graphs, and/or symbols to figure out a plan showing how many brownies he can make.

1 mixing bowl can hold 3 quarts of batter
1 pan holds 1 quart
pan = 9×13
brownie size = 2×2 in²

$$\begin{array}{r} 10 \\ \times 3 \\ \hline 30 \end{array}$$


30 quarts batter

$$\begin{array}{r} 4 \overline{) 30} \\ \underline{12} \\ 18 \\ \underline{16} \\ 20 \end{array}$$

120 brownies

30 quarts = 30 pans

brownie size



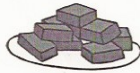
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Teacher Notes:

This student started off with a known piece of information that a mixing bowl would be able to hold 3 quarts of batter. Knowing that there are 10 mixing bowls in the problem scenario, the student multiplied 10 by 3 to arrive at the conclusion that there must be a total of 30 quarts of batter in total. The student estimated that one quart would make a pan, so there must be 30 pans. After figuring this out, the student wrote down that the pan size for baking the brownies would be 9×13 . This was most likely based upon prior knowledge or from measuring the pan provided. I think the student had trouble relating the pan size to a specific number of brownies because the student wrote that each brownie would be 2 inches \times 2 inches without really showing a reason. The student then wrote that only 4 of these brownies would fit into each pan. The student multiplied 30 pans by 4 brownies in each pan to get a total of 120 brownies.

Student work 2

Brownie Batter



Mr. Brown E. Pan recently opened a new business making brownies called "The Brown E. Pan." On his first day baking, he started in his own kitchen by using a single rectangular pan. He has already prepared 10 mixing bowls full of brownie batter.

Flip to back for my answers

Use pictures, words, tables, graphs, and/or symbols to figure out a plan showing how many brownies he can make.

How big is the pan? $L = 13 \text{ in} \cdot W = 9 \text{ in} \cdot H = 2 \text{ in} = 234 \text{ inches}^3$

How much brownies will fit in pan? 12 brownies (with a little extra left)

How big is the mixing bowl? $261 \frac{2}{3} \text{ inches}^3$

Sphere formula: $\text{Volume} = \frac{4}{3} \pi r^3$ $R = 5 \text{ inches}$

Why? $\frac{4}{3} = 1.33$ $1.33 \times 3.14 \times 125 = 523 \frac{1}{3}$ but it is 3×90

How much batter in a bowl? full

How big are brownies? $3 \text{ inch} \cdot 3 \text{ inch} \cdot 2 \text{ inch}$

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$234 \overline{) 2620} \begin{array}{r} 11 \\ - 234 \\ \hline 280 \\ - 234 \\ \hline 46 \\ - 46 \\ \hline 0 \end{array}$

11^{Pans} about

$\begin{array}{r} 11 \\ \times 12 \\ \hline 22 \\ + 110 \\ \hline 132 \end{array}$

132 Brownies

Teacher Notes:

This student started off by finding the size of the pan, most likely by measuring or by using prior knowledge, to be 13 inches x 9 inches x 2 inches. The student found that each pan has a total volume of 234 cubic inches. After figuring this out, the student then tried to figure out how much batter would fit into each mixing bowl. To do this, the student used the volume of a sphere and divided it by 2 to get a mixing bowl. The student estimated the radius of the bowl to be 5 inches, which gave each bowl a total volume of $261 \frac{2}{3}$ cubic inches. The student multiplied this number by 10 to account for all 10 bowls and came up with a total batter amount of 2620 cubic inches. The student took the total amount of batter and divided by the amount of batter in each pan to figure out that there would be about 11 total pans. By determining the size of each brownie to be 3 inches x 3 inches x 2 inches, the student figured out that 12 brownies would fit into each pan for a total of 132 brownies. This student did also indicate in his/her work that there would be some brownie batter that would be left over or possibly wasted.

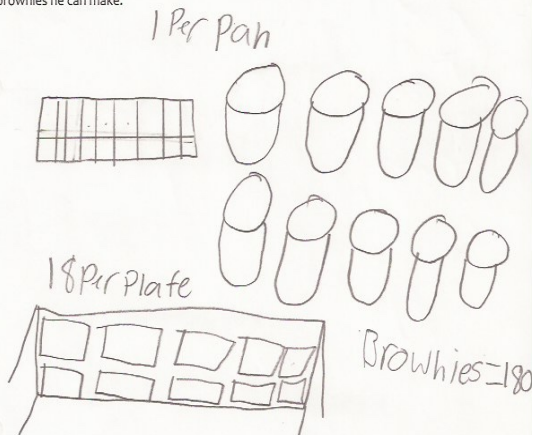
Student work 3

Brownie Batter



Mr. Brown E. Pan recently opened a new business making brownies called "The Brown E. Pan." On his first day baking, he started in his own kitchen by using a single rectangular pan. He has already prepared 10 mixing bowls full of brownie batter.

Use pictures, words, tables, graphs, and/or symbols to figure out a plan showing how many brownies he can make.



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Teacher Notes:

This student started off by drawing each of the 10 mixing bowls. Although it doesn't seem to have influenced the solution, it is interesting to note that the student drew each of these as cylinders. The student indicated on his/her paper that each mixing bowl has enough batter to make one pan of brownies. The student then drew an illustration of the potential baking pan to show that 18 brownies would fit in every pan that is baked. This doesn't seem to be based upon any measurements shown on the paper, so it is most likely an estimate. Once this was figured out, the student only had to multiply the number of brownies in each pan (18) by the total number of brownie pans baked (10) to get a total number of brownies (180). This solution was figured out without using any specific measurements.

Sequencing

- The activity focuses on two main pieces: How many brownies should fit in one pan, and how many pans can be made with 10 mixing bowls of brownie batter.
 - The first portion of student responses will focus on determining how many brownies should be in each pan.
 - The second portion would focus on how we can determine how many pans can be made with 10 mixing bowls of brownie batter.
- Three groups would present their understandings of how many brownies would fit in a single pan.
 - One group will show a drawing or diagram drawn to show how the brownies would fit together in a pan. The group will talk about how they determined that this amount of brownies would make sense because of the size of the brownies.
 - A second group will show a strategy where they will only determine the length and width of brownies in the pan to find the area of how many would fit in a single pan. They will also discuss how size would affect how they made their decision.
 - A third group will be chosen that did something different from the first two groups. This will be a group that chose to make the brownies much smaller than the others. We will discuss how this is different from the other students, and we will discuss how this changes the situation.
- At least four groups will present their findings for determining how many pans could be filled given the 35 gallons of brownie batter.
 - The first group will be one that solved the problem in the most generic way. They will be a group that used more of an estimation approach to figure out how much volume would fill a single pan as well as how many pans of brownies could be made with an entire gallon. They will be a group that uses the tools only sparingly

but still comes up with an answer that gives us a benchmark to start the discussion.

- The second group will be one that used the tools given: the measuring cups and the pans. They will discuss how they determined how much would fit into a single pan, and they will discuss how this helped them to figure out how many total pans could be made.
- The third group will make a connection to linear volume of a rectangular prism. They will discuss how they can use a ruler to measure the volume of batter that would be needed. They will then discuss how this would compare to capacity with standard and metric units. This group will discuss how they used this idea to determine how many pans could be filled.
- A fourth group will be given time to discuss a strategy that is unique and different from most of the other groups. Time should be given to a group that solved the problem in a way that uses the strategies similar to those above, but at the same time, uses a unique idea. This group will be asked questions to compare how their ideas are similar to the other groups.