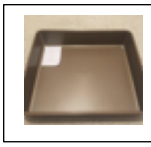
Bite Size Brownies



Designed by:
Jonathan Thompson
George Mason University, COMPLETE Math

The Task

Mr. Brown E. Pan recently opened a new business making brownies called "The Brown E. Pan." On his first day baking, he started in his own kitchen by using the rectangular pan below.

1. Draw a picture to show how many brownies would fill the pan in the picture above.
2. Since he wanted to make the most of his time, he wondered, "What if I make the brownies smaller and add another row of brownies?" How many brownies would now fit into the pan? What if he added another row?
3. Continue the pattern to find the x stage. Find a way to record your results.
4. If the pan held 120 brownies, how many times did Mr. Brown E. Pan add a new row?
5. If he continues this pattern, would it be possible for a pan to hold 500 brownies? Explain how you know.

Big Ideas

- Using perimeter, area, and volume
- Finding relationships between perimeter, area, and volume
- Measurement

Standards of Learning for Grades 3-4-5

- 3.9 – The student will estimate and use U.S. Customary and Metric Units to measure:
  - Length
  - Liquid Volume
  - Weight/Mass
  - Area and Perimeter
- 4.6 - The student will
  - Estimate and measure weight/mass and describe the results in U.S. Customary and Metric Units
  - Identify equivalent measurments between units
- 4.7 - The student will
  - estimate and measure length and describe the result in both Metric and U.S. Customary

Standards of Learning for Grades 6-7-8

- 6.9 – The student will make ballpark comparisons between measurements in the U.S. Customary system and the metric system.
- 6.10 – The student will
  - Solve practical problems involving area and perimeter
  - Describe and dermine the volume of a rectangular prism
- 7.5 – The student will
  - Describe volume of cylinders
  - Solve practical problems involving the volume of rectangular prisms and cylinders
  - Describe how changing one measured attribute of a rectangular prism

○ Identify equivalent measurments between units • 5.8 – The student will ○ Find perimeter, area, and volume in standard units of measure ○ Differentiate between perimeter, area, and volume ○ Identify equivalent measurments within the metric system ○ Choose an appropriate unit of measure for a given situation	affects its volume • 8.7 – The student will ○ Investigate and solve practical problems involving volume of prisms and cylinders ○ Describe how changing one measured attribute of a figure affects the volume and surface area

Process Goals

- Problem Solving and Reasoning – Students will extend a pattern finding the area of a pan of brownies if the length of the pan is continually increased by one brownie.
- Connections and Representations – Students will evaluate a pattern to find how many brownies would be in pan if the length is increased by one brownie over and over, and they will use their knowledge of area and perimeter to find a way to generalize the pattern that they are noticing, using words or variables.
- Communication – Students will use mathematical language to justify their finding snd discuss solution pathways with their peers.

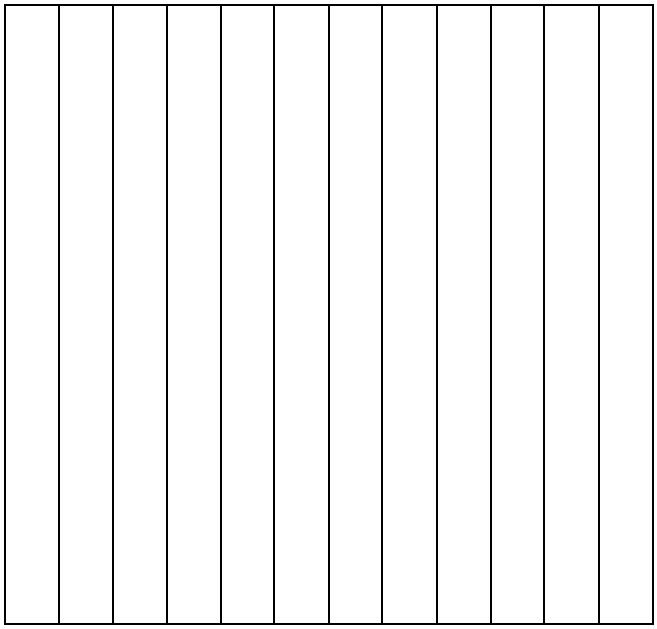
Related Task – Brownie Batter

Mr. Brown E. Pan recently opened a new business making brownies called "The Brown E. Pan." On his first day baking, he started in his own kitchen by using a single rectangular pan. He has already prepared 10 mixing bowls full of brownie batter.

Use pictures, words, tables, graphs, and/or symbols to figure out a plan showing how many brownies he can make.

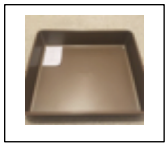
Related Task – Brownie Pan

Mr. Brown E. Pan recently opened a new business making brownies called "The Brown E. Pan." On his first day baking, he started in his own kitchen by using a single square pan. He just finished his first batch of brownies, and he wants to make sure that all the brownies are the same size. He first cut the brownies vertically as shown in the picture below.



The pan of brownies above is cut into 12 congruent rectangles. If the perimeter of each of the rectangles is 65 cm., what is the area of the pan?

Bite Size Brownies
Lesson Plan



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The Task

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1. Draw a picture to show how many brownies would fill the pan in the picture above.
2. Since he wanted to make the most of his time, he wondered, "What if I make the brownies smaller and add another row of brownies?" How many brownies would now fit into the pan? What if he added another row?
3. Continue the pattern to find the x stage. Find a way to record your results.
4. If the pan held 120 brownies, how many times did Mr. Brown E. Pan add a new row?

If he continues this pattern, would it be possible for a pan to hold 500 brownies? Explain how you know.

Materials

- Task sheet for each student
- Task to project on board with pictures of the pan with only one brownie
- Graph paper (used to draw pans with brownies)
- Colored Pencils
- Square manipulatives (paper or tile blocks)
- Presentation paper for each group

Facilitating Task

- Class will be distributed the problem sheet which will be read together and clarifying questions will be answered.
- Student will be split into groups of 3-4 students.
- Before groups work together, each student will be given 5-7 minutes of independent time to begin working on ideas of their own.
- Independent work will transition to small group work (within the 3-4 person groups)
- Groups will be distributed materials when requested
- Groups present findings based upon strategies chosen in order by teacher.

Misconceptions

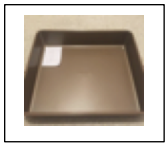
- Brownies are the same size in each individual pan
- Difference between length and width
- Difference between rows and columns
- Pan is staying at a constant size while brownies are changing size

Suggested Prompts or Questions

- How might you draw what you are seeing?
- If there are 4 rows and 3 columns, how many rows would there be if we added another one? How would this affect how many brownies are in the pan?

• Adding to both rows and columns with each successive pan instead of only the rows.	• How would the size of the brownies change? • What is area? Could you use that here to help you find the number of brownies? • If you add another row, how many more brownies does that seem to add? • What pattern do you see that you could apply to any situation? Would it work for what we have already seen? • How could we find out how many brownies would fit the length of the pan? The width? • What do you know about the size of the brownies? How do the first and last brownies compare?

Bite Size Brownies



Name_____

Date_____

Mr. Brown E. Pan recently opened a new business making brownies called “The Brown E. Pan.” On his first day baking, he started in his own kitchen by using the rectangular pan below.

1. Draw a picture to show how many brownies would fill the pan in the picture above.

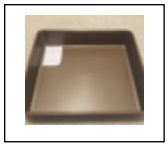
2. Since he wanted to make the most of his time, he wondered, “What if I make the brownies smaller and add another row of brownies?" How many brownies would now fit into the pan? What if he added another row?

3. Continue the pattern to find the x stage. Find a way to record your results.

4. If the pan held 120 brownies, how many times did Mr. Brown E. Pan add a new row?

5. If he continues this pattern, would it be possible for a pan to hold 500 brownies? Explain how you know.

Bite Size Brownies
Anticipation Guide



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The Task

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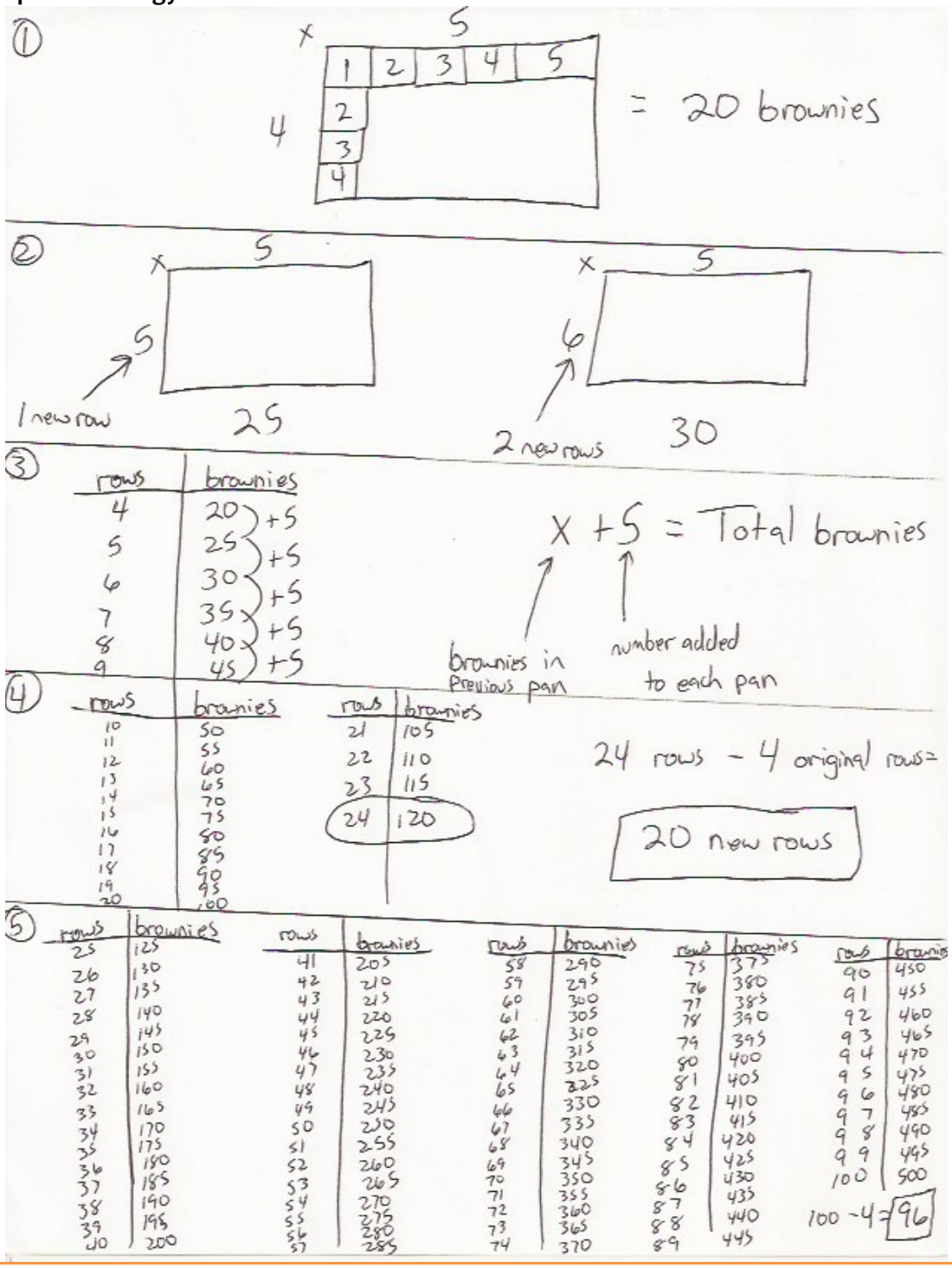
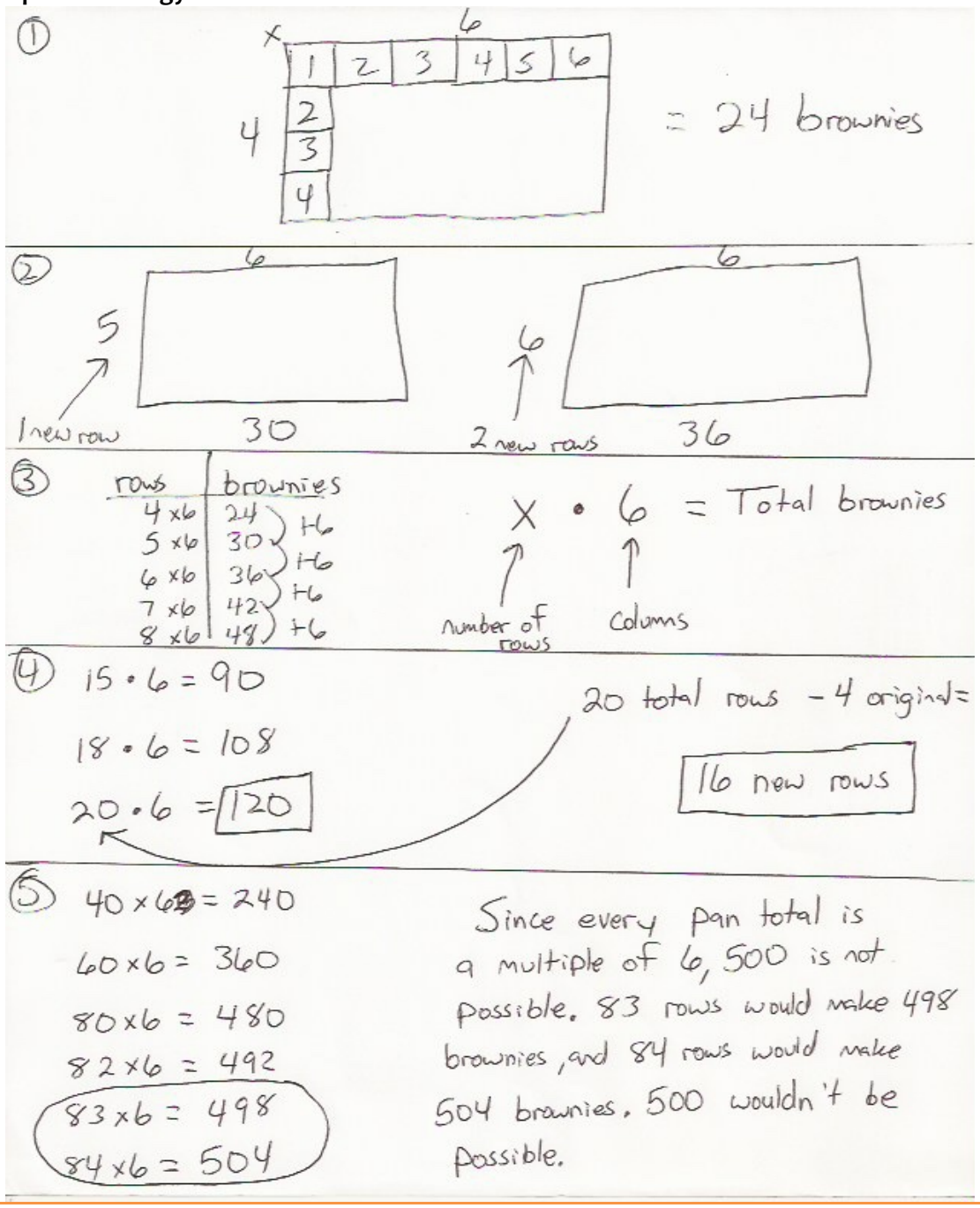
Anticipated Strategy #1



Description:

Students could solve this problem by first finding the amount of brownies in the initial pan. Once they have done this, they would find out how many brownies would be in a pan if one more row would be added and the same for two additional rows. One way to see the pattern is that the amount of brownies in each row (in this case 5) would be added to the total in the pan with each added row. This pattern is shown for number 3 above. After doing this, the student could figure out how many additional rows would need to be added to make 120 and 500 brownies in a pan by extending this pattern upwards. Even though it is quite time consuming, it does yield a correct solution. The student would need to remember to subtract the original number of rows, since the question is asking for the number of rows added.

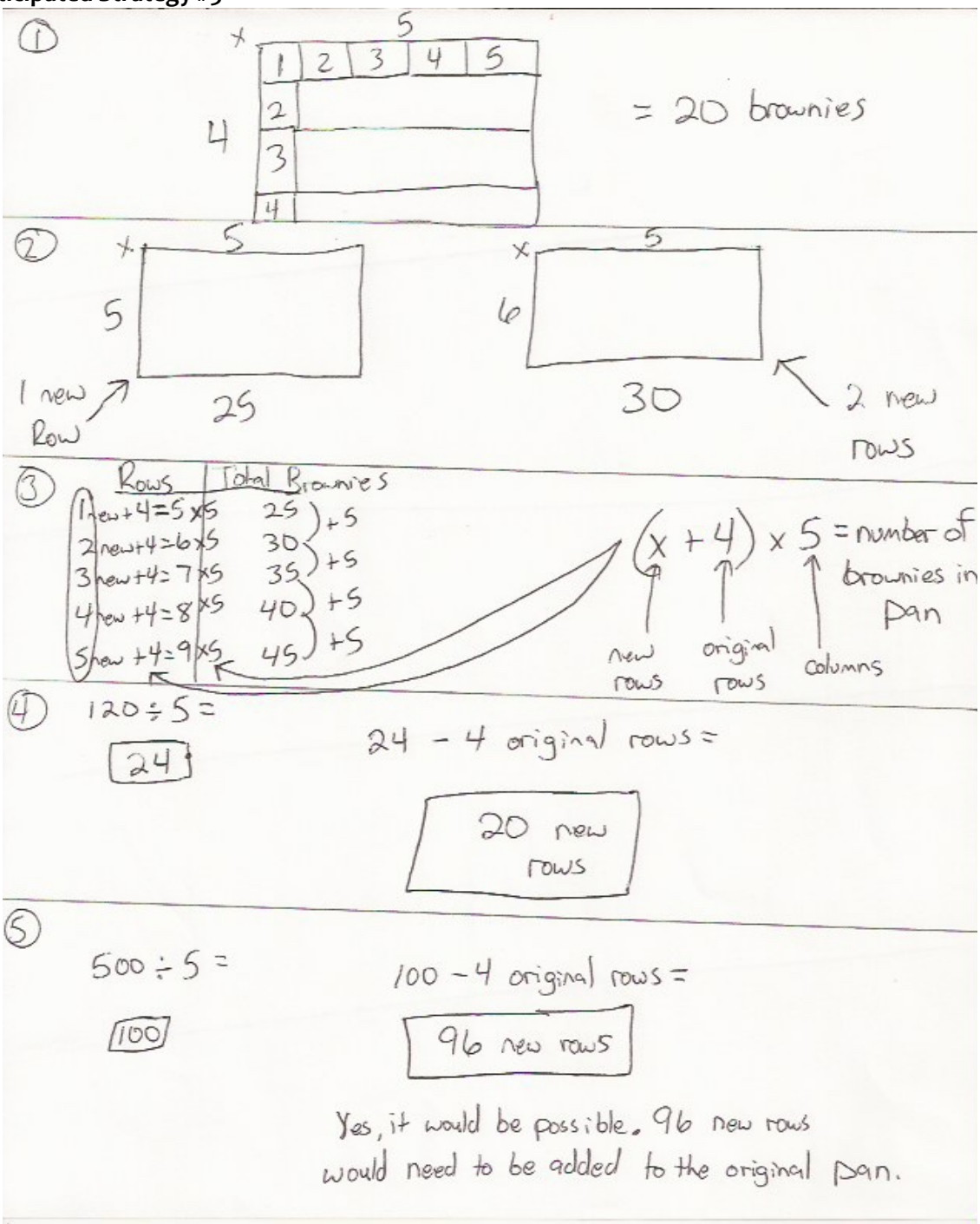
Anticipated Strategy #2



Description:

Students could see the initial pan as having 6 brownies in each row instead of 4 or 5. This will have a big effect on the problem as question 5 comes around. Once this is done, the student would find the total number if an additional row is added and if two additional rows are added. The student could organize data in a t-chart to see that the total number of rows can simply be multiplied by the number of brownies in each row (in this case 6). This would create an equation of the total number of rows multiplied by 6 = the total number of brownies in the pan. To figure out how many rows would need to be added to make 120 brownies in the pan, the student could simply guess and check by multiplying different numbers by 6 until 120 is achieved. This number would have to be decreased by the original number of rows, since the question is asking for the number of additional rows. The same approach can be done to figure out how many additional rows would need to be added to make 500 total brownies. In this case, 500 would not be possible because it is not a multiple of 6.

Anticipated Strategy #3



Description:

Students could start by drawing the pan to determine the number of rows and columns. This could be used to find the total number of brownies. The student could then find the number of brownies if an additional row is added and then if two additional rows are added to the original pan. The student could continue the pattern by seeing that for each additional row that is added, there are four original rows. In each row, there are, in this case, 5 brownies. This would tell the student that they could take the sum of the number of the new rows and the number of original rows. They can then multiply this number by the number of brownies in each row (in this case 5). This could be represented as (x + 4) x 5 = The total number of brownies. To figure out how many additional rows would need to be added to make 120 or 500 brownies in a pan, the student could simply divide the total number by 5 and then subtract by 4 (the number of original rows).

Bite Size Brownies
Student work

Designed by:
Jonathan Thompson
George Mason University, COMPLETE Math
Fall 2016

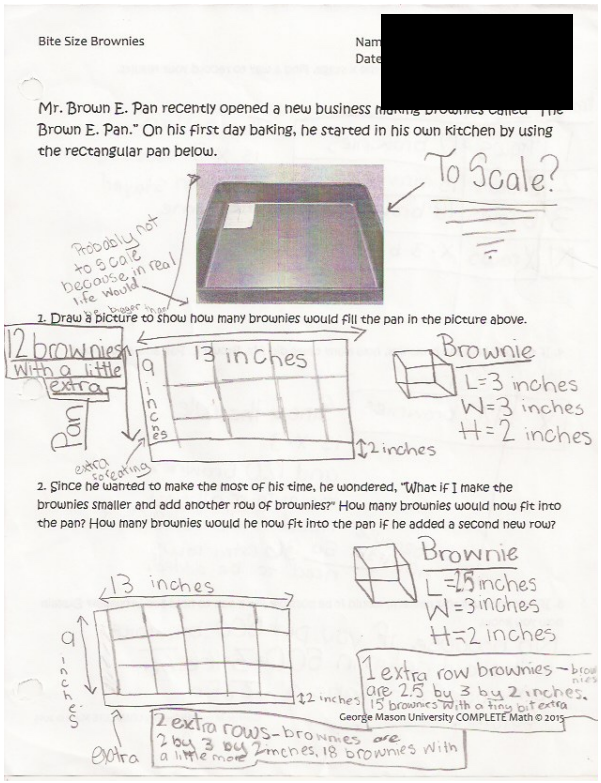
The Task

Mr. Brown E. Pan recently opened a new business making brownies called "The Brown E. Pan." On his first day baking, he started in his own kitchen by using the rectangular pan below.

1. Draw a picture to show how many brownies would fill the pan in the picture above.
2. Since he wanted to make the most of his time, he wondered, "What if I make the brownies smaller and add another row of brownies?" How many brownies would now fit into the pan? What if he added another row?
3. Continue the pattern to find the x stage. Find a way to record your results.
4. If the pan held 120 brownies, how many times did Mr. Brown E. Pan add a new row?

If he continues this pattern, would it be possible for a pan to hold 500 brownies? Explain how you know.

Student work 1



Bite Size Brownies

Name [redacted]
Date

Mr. Brown E. Pan recently opened a new business making brownies called "The Brown E. Pan." On his first day baking, he started in his own kitchen by using the rectangular pan below.

To Scale?

Probably not to scale because in real life would

1. Draw a picture to show how many brownies would fill the pan in the picture above.

12 brownies with a little extra

Pan

9 inches

13 inches

2 inches

extra not fitting

Brownie
L=3 inches
W=3 inches
H=2 inches

2. Since he wanted to make the most of his time, he wondered, "What if I make the brownies smaller and add another row of brownies?" How many brownies would now fit into the pan? How many brownies would he now fit into the pan if he added a second new row?

13 inches

9 inches

2 inches

extra

Brownie
L=2.5 inches
W=3 inches
H=2 inches

1 extra row brownies – brownies are 2.5 by 3 by 2 inches. 15 brownies with a tiny bit extra

2 extra rows-brownies are 2 by 3 by 2 inches, 18 brownies with a little more

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3. Continue the pattern to find the x stage. Find a way to record your results.

trayf#	In	Out
1	4 rows	12 brownies
2	5 rows	15 brownies
3	6 rows	18 brownies
N	X rows	x·3 brownies

The pattern is x·3 because the width stayed the same

4. If the pan held 120 brownies, how many times did Mr. Brown E. Pan add a new row?

In	Out
X	120 brownies

Since the rule is x·3 and 120 brownies is the output, all you need to do is 120÷3. because 40-4=36 so 36 extra rows need to be added.

5. If he continues this pattern, would it be possible for a pan to hold 500 brownies? Explain how you know.

No because if you put 500 brownies in the out column 500÷3=166.$\overline{66}$ and you can't make a .$\overline{66}$ of a column.

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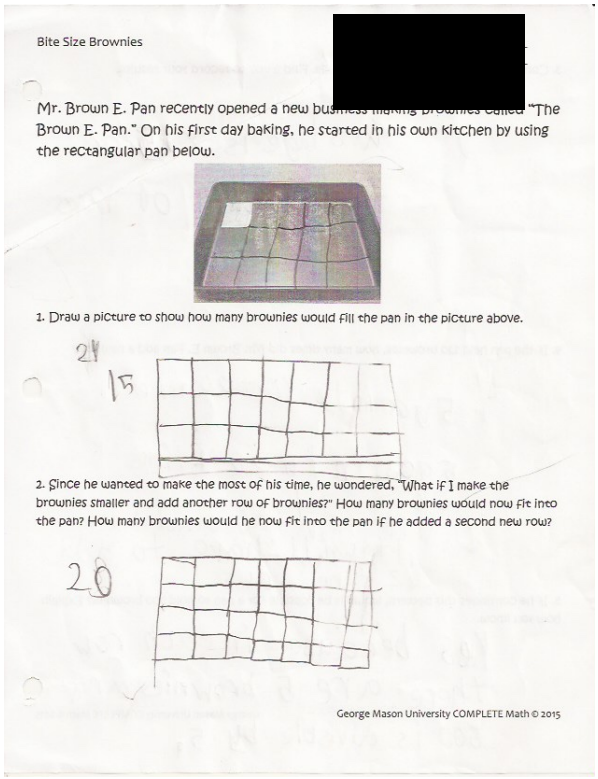
Teacher Notes:

This student worked through the problem by thinking of the pan as a 9 inch x 13 inch pan. The student figured out that 12 brownies would fit (3 x 4) with a little bit extra left over in the pan. When the student added on additional parts to the pan, the student added on an additional column every single time instead of a row. This is misunderstanding that should be addressed multiple times at the beginning of the problem to make sure the students know the difference between the two. The student saw that as more rows were added, more brownies were in the pan. The student also identified that the size of the brownies continued to get smaller.

The student generalized the pattern to be x multiplied by 3, with x being the number of rows and 3 being the width of the pan. The student found the number of rows needed to make 120 brownies by dividing 120 by 3 and then subtracting 4 because the problem asked for the number of additional rows. The student tried to do the same for 500, but noticed that 500 is not divisible by 3. This means that 500 brownies is not possible in this scenario.

Student work 2



Bite Size Brownies

[redacted]

Mr. Brown E. Pan recently opened a new business making brownies called "The Brown E. Pan." On his first day baking, he started in his own kitchen by using the rectangular pan below.

1. Draw a picture to show how many brownies would fill the pan in the picture above.

24 15

2. Since he wanted to make the most of his time, he wondered, "What if I make the brownies smaller and add another row of brownies?" How many brownies would now fit into the pan? How many brownies would he now fit into the pan if he added a second new row?

20

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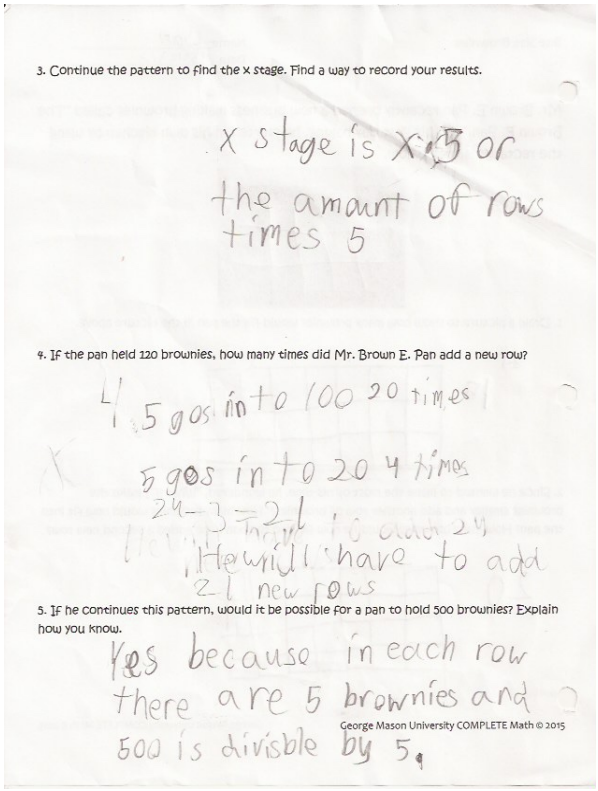

3. Continue the pattern to find the x stage. Find a way to record your results.

x stage is x5 or the amount of rows times 5

4. If the pan held 120 brownies, how many times did Mr. Brown E. Pan add a new row?

5 goes into 100 20 times

5 goes into 20 4 times

24-3=21 I'd add 24. How will I have to add 21 new rows

5. If he continues this pattern, would it be possible for a pan to hold 500 brownies? Explain how you know.

Yes because in each row there are 5 brownies and 500 is divisble by 5.

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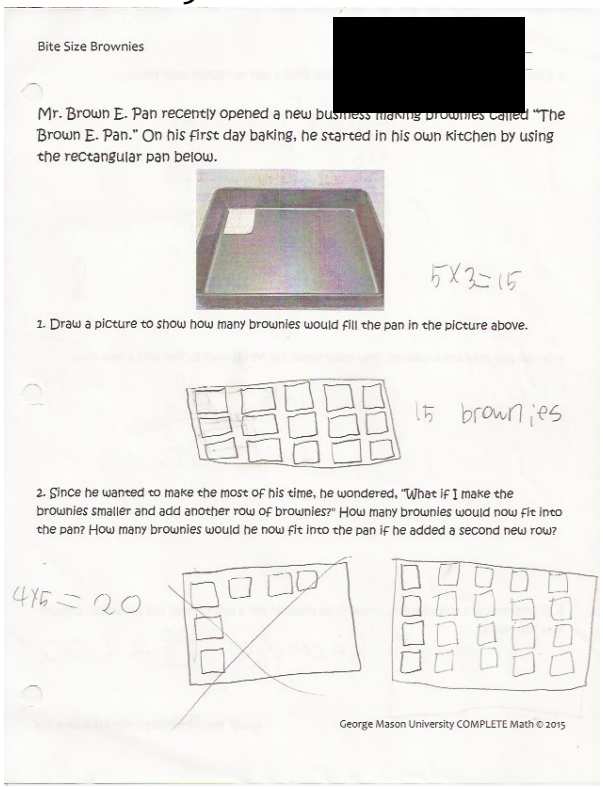
Teacher Notes:

This student started off by drawing the brownies in the original pan. The student figured out that there were 5 columns of brownies and 3 rows of brownies in the original pan, making a total of 15 brownies. The student then showed a picture of what the pan would look like if an additional row were added making the pan still 5 columns long but now 4 rows wide for a total of 20 brownies. The student quickly saw the relationship as being x multiplied by 5, with x being the total number of rows in the pan and 5 being the number of brownies in each row. The student didn't organize data in any way but instead, jumped straight to the conclusion of the x-stage. This was most likely done mentally as the student noticed a pattern of added by 5 repeatedly.

After the student had generalized, the student figured out the number of additional rows that would need to be added to make 120 by by using some division. The student decided that 5 goes into 100 20 times, and 5 goes into 20 4 times, making 24 total rows. Since there were 3 rows originally, the student subtracted 3 from 24 to come to a conclusion that 21 additional rows were added. The student used a similar method to figure out that it would be possible to make a pan with 500 brownies, stating that 500 is divisible by 5.

Student work 3



Bite Size Brownies

[redacted]

Mr. Brown E. Pan recently opened a new business making brownies called "The Brown E. Pan." On his first day baking, he started in his own kitchen by using the rectangular pan below.

5 X 3 = 15

1. Draw a picture to show how many brownies would fill the pan in the picture above.

15 brownies

2. Since he wanted to make the most of his time, he wondered, "What if I make the brownies smaller and add another row of brownies?" How many brownies would now fit into the pan? How many brownies would he now fit into the pan if he added a second new row?

4 X 5 = 20

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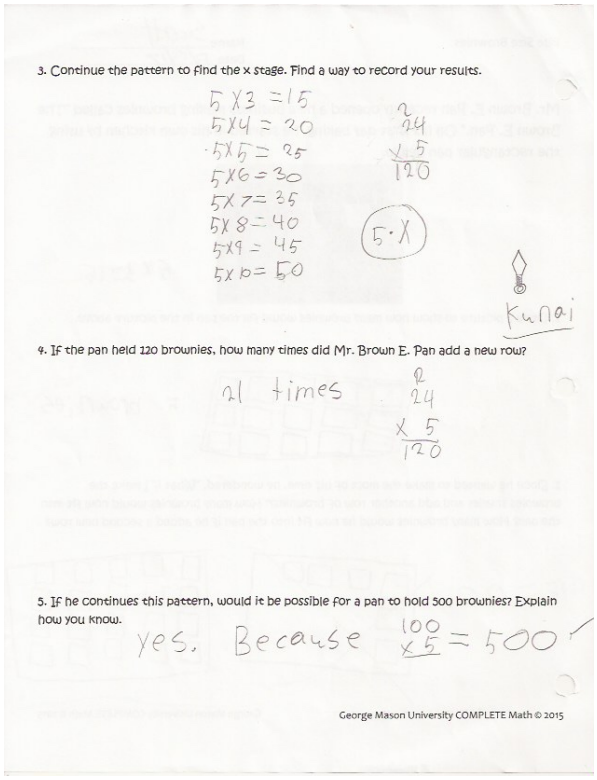

3. Continue the pattern to find the x stage. Find a way to record your results.

5 X 3 = 15
5 X 4 = 20
5 X 5 = 25
5 X 6 = 30
5 X 7 = 35
5 X 8 = 40
5 X 9 = 45
5 X 10 = 50

24
X 5
120

5 · X

Kunai

4. If the pan held 120 brownies, how many times did Mr. Brown E. Pan add a new row?

21 times

24
X 5
120

5. If he continues this pattern, would it be possible for a pan to hold 500 brownies? Explain how you know.

yes. Because 100 x 5 = 500

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Teacher Notes:

This student started by seeing that the original pan had 5 columns and 3 rows, which is illustrated in a picture. This made a total of 15 brownies in the original pan. After seeing this the student drew the next pan that would show one additional row added, making the pan 5 x 4 for a total of 20 brownies. The student didn't draw the pan for two additional rows. To organize data, the student made a column of 5s and a column of the changing number of rows, showing how these two numbers could be multiplied to get the total number of brownies. This is a great way to show what is happening because it shows that the 5 is constant, and the number of rows is what is varying. Through this work, the student came to a conclusion that the x-stage could be represented as 5 multiplied by x, where x is the number of rows in each pan.

To figure out the amount of additional rows it would take to make 120 brownies in the pan, the student multiplied 24 by 5 to make 120. The student most likely knew this product because it is the only problem shown. The student wrote an answer of 21, most likely because he/she subtracted the three additional rows from 24. The student used a similar method for the last question by simply saying that 100 x 5 = 500, so 500 must be possible.

Sequencing

- Since the problem focuses on two main problems, I would sequence the student response portion into two pieces.
    - The first portion of student responses would focus on how many brownies were in the original pan.
    - The second portion would focus on the pattern when various numbers multiply the length and width.
- Three groups would present their understandings of how the brownie pan would look filled.
    - One group would show how they drew out all the brownies in the pan to show a completed pan. Whether they are exactly right or not, their response will feature a drawing method to figure out this portion of the problem.
    - A second group will show how they only figured out the length and width of the pan using brownies and then multiplied these to get the total amount. They will show the connection between how they found they also filled in parts of the pan, but they will discuss why they didn't need to fill in the whole pan like the previous group.
    - A third group will use a similar method to the second group, but they will use the term area. They will talk about how they took the length and width as the second group did but further defined the concept.
- At least four groups will present their findings for the pattern extension portion of the problem.
    - The first group will focus on the idea that the number of rows will be increased by 1. They will show how this changed the number of brownies from the original pan. They will show how they found the new length and width and then multiplied to find the area.
    - The second group will focus more on the added sections. They will look at the original section, and they will color the added sections in a different color. They will show how their understandings are the same as the original team, but they will add to this by showing a different way to see the whole amount of brownies each time. This will help everybody to see exactly how many brownies are being added with each successive row.
    - The third group will move forward in extending the pattern by showing how there is a multiplication pattern each time. They will show how the number of rows in each pan can simply be multiplied by the number of brownies in each row. They will show this at first with a picture but then only with a chart. They will show quickly how the number of brownies would change if a few rows are added.
    - The last group will take the previous group's ideas and generalize them. They will show how the idea could work for any given situation, including the x stage.